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Understand Place Value

The number 391,568 may be easier to read and write if you use a place-value chart.

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 0 0 0 ,</td>
<td>0 0 0</td>
</tr>
<tr>
<td>9 0 ,</td>
<td>0 0 0</td>
</tr>
<tr>
<td>1 ,</td>
<td>0 0 0</td>
</tr>
<tr>
<td>, 5 0 0</td>
<td></td>
</tr>
<tr>
<td>3 9 1 ,</td>
<td>5 6 8</td>
</tr>
</tbody>
</table>

**Standard form:** 391,568  
**Expanded form:** 300,000 + 90,000 + 1,000 + 500 + 60 + 8  
**Word form:** Three hundred ninety-one thousand, five hundred sixty-eight

Write the number in the place-value chart. Then write the number in expanded form.

1. 716,583
2. 78,056

Use the place-value chart to help you write the value of the bold-faced digit.

3. 58,346
4. 723,308

5. 468,005
6. 420,822
Millions and Billions

You can use a place-value chart to help you read and write greater numbers such as 721,306,984.

<table>
<thead>
<tr>
<th>Millions</th>
<th>Thousands</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>T</td>
<td>O</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Expanded form: $700,000,000 + 20,000,000 + 1,000,000 + 300,000 + 6,000 + 900 + 80 + 4$
Word form: seven hundred twenty-one million, three hundred six thousand, nine hundred eighty-four

Write the number in word form.
1. 2,267,025,142
2. 702,326,500

Write the number in standard form.
3. $600,000,000 + 50,000,000 + 9,000,000 + 800,000 + 40,000 + 3,000 + 700 + 1$
4. thirty-five billion, eight hundred six million, four hundred eighty-six thousand, two hundred twenty-six
Compare Numbers

You can use a place-value chart to compare numbers.

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 8 9 5</td>
<td>2 8 9 5</td>
</tr>
</tbody>
</table>

Compare the digits from left to right.

First number: 289,865
Second number: 289,765

So, 289,865 > 289,765.

Complete the place-value chart. Write <, >, or = for each .

1. 375,841 367,841
2. 677,860 677,860
3. 467,935 476,935
4. 986,496 986,495
5. 47,206,385 47,083,219
**Order Numbers**

You can use a place-value chart to order numbers. Compare the digits from left to right.

Since 4 > 2, 342,198 is the greatest number.

Continue to compare with the remaining two numbers.

Since 6 > 5, 322,678 > 322,501.

---

Use the place-value chart to order the numbers.

1. 144,421; 144,321; 145,221

2. 532,124; 58,124; 532,876

3. 456,342,523; 456,342,876; 494,123,563
Problem Solving Skill

Use a Table

Tables help organize data so you can make comparisons.

Suppose you want to compare the sizes of four planets.
You could make the following table.

<table>
<thead>
<tr>
<th>THE PLANETS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Diameter (miles)</td>
</tr>
<tr>
<td>Mercury</td>
<td>3,030</td>
</tr>
<tr>
<td>Venus</td>
<td>7,517</td>
</tr>
<tr>
<td>Earth</td>
<td>7,921</td>
</tr>
<tr>
<td>Mars</td>
<td>4,222</td>
</tr>
</tbody>
</table>

- Look at the diameters. Compare the digits from left to right.
- The smallest planet is Mercury. The largest planet is Earth.

Use the tables to answer the questions.

1. This table shows the sales for a popular music store chain. Which type of music had the greatest sales amount? the least sales amount?

<table>
<thead>
<tr>
<th>MUSIC SALES</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Music Type</td>
<td>Sales (in dollars)</td>
</tr>
<tr>
<td>-------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>Alternative Rock</td>
<td>1,345,850</td>
</tr>
<tr>
<td>Classical</td>
<td>548,290</td>
</tr>
<tr>
<td>Country</td>
<td>1,930,000</td>
</tr>
<tr>
<td>Light Rock</td>
<td>425,830</td>
</tr>
</tbody>
</table>

2. This table shows the areas of some of the world’s oceans. Which of these oceans has the greatest area? the least area?

<table>
<thead>
<tr>
<th>OCEANS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Area (square miles)</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------</td>
</tr>
<tr>
<td>Indian</td>
<td>31,507,000</td>
</tr>
<tr>
<td>North Pacific</td>
<td>32,225,000</td>
</tr>
<tr>
<td>South Pacific</td>
<td>25,298,000</td>
</tr>
<tr>
<td>North Atlantic</td>
<td>18,059,000</td>
</tr>
<tr>
<td>South Atlantic</td>
<td>14,426,000</td>
</tr>
</tbody>
</table>
Tenths and Hundredths

Money can be used to model decimals.

A dollar represents one whole, or $1.00.

The whole is divided into 10 equal parts.
One dime is \( \frac{1}{10} \) of a dollar, or $0.10.

The whole is divided into 100 equal parts.
One penny is \( \frac{1}{100} \) of a dollar, or $0.01.

Write as a decimal.

1. 2. 3.
4.

1. 2. 3.
4.

4. 1 dollar, 2 dimes, and 9 pennies
5. 3 dollars and 6 dimes
6. 7 dollars, 5 dimes, and 7 pennies

Write as a decimal and a fraction.

7. 4 dimes and 6 pennies
8. 2 dollars and 7 dimes
9. 3 dollars, 6 dimes, and 5 pennies

10. five and six tenths
11. five hundredths
12. four and three tenths

13. one and eighty-three hundredths
14. nine and seventeen hundredths
15. two and eight tenths
Thousandths and Ten-Thousandths

A place-value chart can help you find the value of each digit in a decimal.

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
<th>Ten-Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>6</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Read: two three tenths six hundredths five thousandths one ten-thousandth

Write: 2.0 0.3 0.06 0.005 0.0001

In Standard Form: 2.3651
In Expanded Form: $2.0 + 0.3 + 0.06 + 0.005 + 0.0001$
In Word Form: two and three thousand, six hundred fifty-one ten-thousandths

Record each decimal in the place-value chart. Write each decimal in expanded form and word form.

1. 1.5138

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
<th>Ten-Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. 4.973

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
<th>Ten-Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. 7.0458

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
<th>Ten-Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Equivalent Decimals**

**Equivalent decimals** are different names for the same number or amount.

- 2 tenths = 20 hundredths
- 0.2 = 0.20

In the place-value chart, both numbers have a 2 in the tenths place.

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

The zero to the right of the 2 does not change the value of the decimal. So, they are equivalent.

Write the numbers in the place-value chart. Then write **equivalent** or **not equivalent** to describe each pair of decimals.

1. 2.5 and 2.50
2. 0.73 and 0.703

Write the two decimals that are equivalent.

3. 3.05
4. 1.110
5. 0.180
6. 7.77

Write an equivalent decimal for each number.

7. 0.05
8. 2.100
9. 2.875
10. 0.040
Compare and Order Decimals

You can use a place-value chart to compare 6.741 and 6.742.

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>7</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

6.741

So, 6.742 > 6.741.

Write the numbers in the place-value chart. Then write <, >, or = in each circle.

1. 2.45 □ 2.54
2. 6.23 □ 6.230
3. 72.648 □ 72.658
4. 564.876 □ 564.786

Write <, >, or = in each circle.

5. 3.21 □ 3.210
6. 721.460 □ 72.146
7. 6.275 □ 6.257
8. 468.036 □ 468.136

Order from least to greatest.

9. 16.54, 16.56, 16.55
10. 3.400, 3.004, 3.040
**Problem Solving Skill**

**Draw Conclusions**

Michael exercises at 4:00 P.M. daily unless he is sick. The table shows the number of hours Michael exercised last week.

Can the conclusion be drawn from the information given? Write **yes** or **no**. Explain your choice.

<table>
<thead>
<tr>
<th>Day</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>1.8</td>
</tr>
<tr>
<td>Tuesday</td>
<td>1.5</td>
</tr>
<tr>
<td>Wednesday</td>
<td>0</td>
</tr>
<tr>
<td>Thursday</td>
<td>2.2</td>
</tr>
<tr>
<td>Friday</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Michael usually eats dinner at 5:30.  
Michael was sick on Wednesday.

Can the conclusion be drawn from the information given? Write **yes** or **no**. Explain your choice.

During a kickball game between two gym classes, the final score was 25 to 22. Each team had 15 players.

1. There were more boys on the winning team than on the losing team.  
   2. There was a winning team.

3. Each player kicked a homerun.  
   4. More than 40 points were scored in the game.
Round Whole Numbers

You can round whole numbers by using the rounding rules.

**Step 1:** UNDERLINE the digit in the place to which you want to round.

**Step 2:** COMPARE the digit to the right of the underlined digit to 5.
- **Round Down:** If the digit to the right is less than 5, the underlined digit stays the same.
- **Round Up:** If the digit to the right is 5 or greater, increase the underlined digit by 1.

**Step 3:** CHANGE all digits to the right of the underlined digit to zeros.

**A.** Round 43,658 to the nearest hundred.
- **UNDERLINE.** 43,658
- **COMPARE.** 5 = 5
- **Round Up.**
- **CHANGE.** 43,700

**B.** Round 9,309,587 to the nearest million.
- **UNDERLINE.** 9,309,587
- **COMPARE.** 3 < 5
- **Round Down.**
- **CHANGE.** 9,000,000

Round each number to the place of the _bold-faced_ digit.

1. 38,761
- **UNDERLINE.** 38,761
- **COMPARE.** ___ ○ 5
- **Round.**
- **CHANGE.**

2. 719,432
- **UNDERLINE.** 719,432
- **COMPARE.** ___ ○ 5
- **Round.**
- **CHANGE.**

Round 2,409,485 to the place named.

3. hundred thousands
- **UNDERLINE.** 2,409,485
- **COMPARE.** ___ ○ 5
- **Round.**
- **CHANGE.**

4. hundreds
- **UNDERLINE.** 2,409,485
- **COMPARE.** ___ ○ 5
- **Round.**
- **CHANGE.**
Estimate Sums and Differences

You can estimate sums and differences by rounding the numbers in the problem before performing the operation. One way to round is to round to the greatest place-value position. For example:

A. Estimate the **sum** by rounding.

\[
\begin{align*}
3,709,525 & \rightarrow 4,000,000 \\
+ 567,802 & \rightarrow + 600,000 \\
\hline
4,600,000 & \\
\end{align*}
\]

The sum is about 4,600,000.

B. Estimate the **difference** by rounding.

\[
\begin{align*}
539,014 & \rightarrow 500,000 \\
- 205,918 & \rightarrow - 200,000 \\
\hline
300,000 & \\
\end{align*}
\]

The difference is about 300,000.

Estimate by rounding.

1. \[
\begin{align*}
473,542 & \rightarrow \\
+ 207,958 & \rightarrow + \_ \_ \_ \_ \_ \_ \\
\hline
\_ \_ \_ \_ \_ \_ & \\
\end{align*}
\]

2. \[
\begin{align*}
741,356 & \rightarrow \\
- 157,900 & \rightarrow - \_ \_ \_ \_ \_ \_ \\
\hline \_ \_ \_ \_ \_ \_ & \\
\end{align*}
\]

3. \[
\begin{align*}
8,619,724 & \rightarrow \\
+ 3,970,685 & \rightarrow + \_ \_ \_ \_ \_ \_ \\
\hline \_ \_ \_ \_ \_ \_ & \\
\end{align*}
\]

4. \[
\begin{align*}
5,101,118 & \rightarrow \\
- 496,007 & \rightarrow - \_ \_ \_ \_ \_ \_ \\
\hline \_ \_ \_ \_ \_ \_ & \\
\end{align*}
\]

5. \[
\begin{align*}
724,581 & \rightarrow \\
- 219,067 & \rightarrow - \_ \_ \_ \_ \_ \_ \\
\hline \_ \_ \_ \_ \_ \_ & \\
\end{align*}
\]

6. \[
\begin{align*}
192,837 & \rightarrow \\
+ 445,672 & \rightarrow + \_ \_ \_ \_ \_ \_ \\
\hline \_ \_ \_ \_ \_ \_ & \\
\end{align*}
\]

7. \[
\begin{align*}
521,739 & \rightarrow \\
+ 659,931 & \rightarrow + \_ \_ \_ \_ \_ \_ \\
\hline \_ \_ \_ \_ \_ \_ & \\
\end{align*}
\]

8. \[
\begin{align*}
911,011 & \rightarrow \\
+ 187,408 & \rightarrow + \_ \_ \_ \_ \_ \_ \\
\hline \_ \_ \_ \_ \_ \_ & \\
\end{align*}
\]

9. \[
\begin{align*}
4,516,361 & \rightarrow \\
+ 3,497,205 & \rightarrow + \_ \_ \_ \_ \_ \_ \\
\hline \_ \_ \_ \_ \_ \_ & \\
\end{align*}
\]

10. \[
\begin{align*}
6,212,345 & \rightarrow \\
- 3,493,968 & \rightarrow - \_ \_ \_ \_ \_ \_ \\
\hline \_ \_ \_ \_ \_ \_ & \\
\end{align*}
\]
Add and Subtract Whole Numbers

You can add or subtract to find an exact answer.

Estimates will help you determine if you have a reasonable answer.

Suppose you have saved 3,857 pennies. Then your mom gives you 2,234 more pennies to help you buy a present for a friend. How many pennies do you have altogether?

First, estimate. Then find the exact sum or difference.

First, estimate. The answer should be close to 6,000. Then, add to find the exact answer. 6,091 is close to the estimate, so the answer is reasonable. You have 6,091 pennies.

Estimate. Then find the exact sum or difference.

1. \[ \begin{array}{c} 5\quad 4\quad 9\quad 2 \\ + \quad 4\quad 0\quad 7\quad 8 \\ \hline \quad 9\quad 5\quad 7\quad 0 \end{array} \]

2. \[ \begin{array}{c} 7\quad 9\quad 0\quad 6 \\ - \quad 4\quad 2\quad 3\quad 4 \\ \hline \quad 3\quad 6\quad 7\quad 2 \end{array} \]

3. \[ \begin{array}{c} 2\quad 9\quad 5\quad 3\quad 6 \\ - \quad 1\quad 0\quad 8\quad 1\quad 9 \\ \hline \quad 1\quad 7\quad 7\quad 1\quad 7 \end{array} \]

4. \[ \begin{array}{c} 6\quad 8\quad 4\quad 4 \\ + \quad 4\quad 7\quad 3\quad 9 \\ \hline \quad 1\quad 4\quad 2\quad 8\quad 3 \end{array} \]

5. \[ \begin{array}{c} 1\quad 3\quad 7\quad 6 \\ - \quad 4\quad 3\quad 2 \\ \hline \quad 7\quad 4\quad 4 \end{array} \]

6. \[ \begin{array}{c} 3\quad 6\quad 7\quad 4\quad 8 \\ + \quad 1\quad 4\quad 2\quad 4\quad 7 \\ \hline \quad 4\quad 1\quad 2\quad 9\quad 5\quad 5 \end{array} \]
Choose a Method

You add and subtract greater numbers the same way you add and subtract smaller numbers.

It may become difficult to keep place values aligned when adding and subtracting greater numbers. Commas help you to line up the numbers.

For example, find the sum of 6,716,678 and 5,014,209.

- Line up the addends along the commas.
  \[
  \begin{array}{c}
  6,716,678 \\
  + 5,014,209 \\
  \hline
  11,730,887
  \end{array}
  \]

- Add to find the exact answer.
  \[
  \begin{array}{c}
  6,716,678 \\
  + 5,014,209 \\
  \hline
  11,730,887
  \end{array}
  \]

- Estimate the sum to see if your answer is reasonable.

11,730,887 is close to the estimate of 12,000,000, so the answer is reasonable.

Find the sum or difference. Estimate to check.

1. \[ \begin{array}{c}
  8,432,790 \\
  + 3,876,339 \\
  \hline
  12,309,129
  \end{array} \]

2. \[ \begin{array}{c}
  4,918,471 \\
  - 1,839,220 \\
  \hline
  3,079,251
  \end{array} \]

3. \[ \begin{array}{c}
  9,010,776 \\
  - 4,573,932 \\
  \hline
  4,436,844
  \end{array} \]

4. \[ \begin{array}{c}
  3,825,449 \\
  + 4,361,749 \\
  \hline
  8,187,208
  \end{array} \]

Copy the problem. Use commas to help you line up the numbers. Find the sum or difference. Estimate to check.

5. \[ \begin{array}{c}
  6,654,148 \\
  + 4,732,387 \\
  \hline
  11,386,535
  \end{array} \]

6. \[ \begin{array}{c}
  7,927,881 \\
  - 4,618,532 \\
  \hline
  3,309,349
  \end{array} \]
Problem Solving Strategy

Use Logical Reasoning

A table can help you with logical reasoning.

Elizabeth, Alan, Calvin, and Marie each ordered a different ice cream flavor. The flavor choices were vanilla, peach, chocolate, and strawberry. Neither Alan nor Marie ordered vanilla. Calvin had a brown ice cream stain on his t-shirt. Marie is allergic to strawberries. Which flavor ice cream did each person order?

- Calvin had a brown stain on his t-shirt. Put a yes in the chocolate column for Calvin and a no in each empty box in that row and in that column.

- Marie is allergic to strawberries and she did not order vanilla. Put a no in those boxes. Put a yes in the remaining box, peach, and a no in the remaining boxes in that column.

- Alan did not order vanilla. Put a no in that box. That leaves strawberry.

- So, Elizabeth ordered vanilla. Put a yes in that box.

<table>
<thead>
<tr>
<th></th>
<th>vanilla</th>
<th>peach</th>
<th>chocolate</th>
<th>strawberry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elizabeth</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Alan</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Calvin</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Marie</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Use logical reasoning and the table to solve.

1. Rishawn, Julie, Kevin, and LaTia each have a different favorite subject. Julie likes to use paint and chalk. LaTia enjoys using numbers. Science is not Kevin’s favorite subject. What is each student’s favorite subject?
Round Decimals

The same rules you learned for rounding whole numbers can be used to round decimals.

**Step 1:** Underline the digit in the place to which you want to round.

**Step 2:** Compare the digit at the right of the underlined digit to 5.
- Round Down: If the digit at the right is less than 5, the underlined digit stays the same.
- Round Up: If the digit at the right is 5 or more, increase the underlined digit by 1.

**Step 3:** Rewrite all digits to the right of the underlined digit as zeros.
An equivalent decimal can be written by leaving off trailing zeros.

<table>
<thead>
<tr>
<th>A. Round 5.6431 to the nearest hundredth.</th>
<th>B. Round 0.8287 to the nearest thousandth.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underline. 5.6431</td>
<td>Underline. 0.8287</td>
</tr>
<tr>
<td>Compare. 3 &lt; 5 Round down.</td>
<td>Compare. 7 &gt; 5 Round up.</td>
</tr>
<tr>
<td>Rewrite. 5.6400 or 5.64</td>
<td>Rewrite. 0.8290 or 0.829</td>
</tr>
</tbody>
</table>

1. Round 4.1872 to the place of the **bold-faced** digit.
   - Underline. 4.1872
   - Compare. ___ 5 Round ____.
   - Rewrite. ________________

2. Round 82.64751 to the nearest thousandth.
   - Underline. 82.6475
   - Compare. ___ 5 Round ____.
   - Rewrite. ________________

Round each number to the place of the **bold-faced** digit.

3. 7.325
   - _______

4. 9.0287
   - _______

5. 108.108
   - _______

6. 26.3199
   - _______

Round 12.8405 to the place named.

7. hundredths
   - _______

8. ones
   - _______

9. tenths
   - _______

10. thousandths
    - _______
**Estimate Decimal Sums and Differences**

Jonas earned $25.87. Kevin earned $20.94. About how much did they earn in all? About how much more did Jonas earn than Kevin?

You can estimate decimal sums and differences by rounding the amounts to the nearest whole number and then adding or subtracting.

<table>
<thead>
<tr>
<th>A. Estimate the sum by rounding.</th>
<th>B. Estimate the difference by rounding.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$25.87 \rightarrow \quad $26</td>
<td>$25.87 \rightarrow \quad $26</td>
</tr>
<tr>
<td>+ 20.94 \rightarrow + 21</td>
<td>- 20.94 \rightarrow - 21</td>
</tr>
<tr>
<td>\text{They earned about $47.}</td>
<td>\text{Jonas earned about $5 more than Kevin.}</td>
</tr>
</tbody>
</table>

Estimate the sum or difference by rounding to the nearest whole number or dollar.

1. $63.98 \rightarrow 
   + 5.29 \rightarrow 
2. 9.684 \rightarrow 
   - 2.395 \rightarrow 
3. 25.39 \rightarrow 
   - 17.71 \rightarrow 

Estimate the sum or difference to the nearest tenth.

4. 8.604 \rightarrow 
   - 6.71 \rightarrow 
5. 26.4572 \rightarrow 
   + 11.3518 \rightarrow 
6. 56.8 \rightarrow 
   + 8.592 \rightarrow 

Estimate the sum or difference.

7. 8.453 \rightarrow 
   - 1.21 \rightarrow 
8. 8.25 \rightarrow 
   + 0.385 \rightarrow 
9. 9.52 \rightarrow 
   + 1.29 \rightarrow 

10. 7.05 \rightarrow 
    - 0.63 \rightarrow 
11. 5.128 \rightarrow 
    - 1.56 \rightarrow 
12. 2.31 \rightarrow 
    + 4.804 \rightarrow 

Reteach RW17
Add and Subtract Decimals

To add or subtract decimals, line up the decimal points in the problem. Finding an estimate first will help you determine if your answer is reasonable.

First, estimate.

\[
\begin{align*}
18.948 & \rightarrow 19 \\
-5.765 & \rightarrow -6 \\
\hline
13 & \rightarrow 13
\end{align*}
\]

Then, subtract to find the exact answer.

\[
\begin{align*}
1 & \quad 8 \quad 9 \quad 4 \quad 8 \\
- & \quad 5 \quad 7 \quad 6 \quad 5 \\
\hline
1 & \quad 3 \quad 1 \quad 8 \quad 3
\end{align*}
\]

The answer should be close to 13.

13.183 is close to the estimate, so the answer is reasonable.

Estimate. Then find the exact sum or difference.

1. \[
\begin{align*}
1 & \quad 5 \\
+ & \quad 5 \quad 3 \\
\hline
\end{align*}
\]

2. \[
\begin{align*}
1 & \quad 8 \quad 5 \quad 2 \\
+ & \quad 3 \quad 7 \quad 3 \\
\hline
\end{align*}
\]

3. \[
\begin{align*}
6 & \quad 3 \quad 9 \\
+ & \quad 7 \quad 8 \quad 5 \\
\hline
\end{align*}
\]

4. \[
\begin{align*}
8 & \quad 7 \quad 6 \\
- & \quad 2 \quad 3 \\
\hline
\end{align*}
\]

5. \[
\begin{align*}
1 & \quad 6 \quad 3 \quad 2 \\
- & \quad 4 \quad 8 \\
\hline
\end{align*}
\]

6. \[
\begin{align*}
6 & \quad 2 \quad 8 \\
- & \quad 9 \quad 6 \\
\hline
\end{align*}
\]

7. \(5.86 + 8.79 = n\)

8. \(14.09 - 2.87 = n\)
Zeros in Subtraction

Find $1.34 - 1.256$.

- To subtract decimal numbers, line up the numbers along the decimal points.
  \[
  \begin{array}{r}
  1.34 \\
  - 1.256 \\
  \hline
  \end{array}
  \]

- Add zeros so both numbers have the same number of decimal places.
  \[
  \begin{array}{r}
  1.340 \\
  - 1.256 \\
  \hline
  \end{array}
  \]

- Subtract.
  \[
  \begin{array}{r}
  1.340 \\
  - 1.256 \\
  \hline
  0.084
  \end{array}
  \]

- Place a decimal point in the answer, below the decimal points in the problem.

So, $1.34 - 1.256 = 0.084$.

Find the difference.

1. $2.7 - 1.5$
2. $3.94 - 2.6$
3. $4.75 - 2.56$
4. $6.8 - 3.9$
5. $5.1 - 3.08$

6. $3.5 - 2.8$
7. $4.4 - 1.65$
8. $7.643 - 3.4$
9. $11.904 - 8.626$
10. $16.24 - 9.1$

11. $4.2 - 2.83$
12. $5.6 - 3.58$
13. $9.41 - 6.527$
14. $14.5 - 8.872$
15. $35.4 - 15.567$

16. $3.84 - 2.68 = n$
17. $2.7 - 0.312 = n$
18. $4.1 - 3.3 = n$

19. $6.57 - 1.898 = n$
20. $5.2 - 2.623 = n$
21. $7.42 - 3.416 = n$
Problem-Solving Skill

Estimate or Find Exact Answer

Both estimation and exact answers are useful when shopping.

Estimations help you determine if you have enough money. Exact answers help you determine if you received the correct change.

Suppose you have $5.00, and want to buy 5 drinks for $0.85 each. Do you have enough money? How much change will you receive?

**Estimation**

| $0.85 → | $1.00 |
| $0.85 → | $1.00 |
| $0.85 → | $1.00 |
| $0.85 → | $1.00 |
| $0.85 → | $1.00 |

**Exact Answer**

<table>
<thead>
<tr>
<th>$0.85</th>
<th>$5.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.85</td>
<td>$4.25</td>
</tr>
<tr>
<td>$0.85</td>
<td>$0.75</td>
</tr>
</tbody>
</table>

So, you have enough money. So, you should receive $0.75 change.

Write an estimate of the total amount. Then solve.

1. Pat has $10.00. He wants to buy a magazine for $3.25, a small pizza for $3.89, and two drinks for $1.15 each. How much change will Pat receive?

2. Paula has $45. She wants to purchase CDs costing $12.99, $14.99, $9.99, and $11.99. Does Paula have enough money? If so, how much change will she receive?

3. Jenny has $10.00. She wants to buy 6 pounds of apples costing $0.75 per pound and a bag of oranges costing $1.45. What is Jenny’s exact cost? How much change will she receive?

4. Erin has $50.00. She wants to buy a purse for $17.99, gloves for $10.98, and a sweater for $19.95. What is her exact cost? How much change will Erin receive?
Expressions and Variables

An expression has numbers and operation signs. It does not have an equal sign.

Use these words to help you write expressions.

**Addition:** more, sum, plus, added, gave

**Subtraction:** less, minus, loss, difference, spent, left

John had 12 marbles. He won 7 more.  
Mary had $10. She spent $3.  
Translate this into an expression.  
Translate this into an expression.  
Clue Word: more \( 12 + 7 \)  
Clue Word: spent \( 10 - 3 \)

An expression may have a variable. A variable is a letter or symbol that can stand for a number.

Peter caught 2 fish in the morning.  
In the afternoon, he caught some more.  
Translate this into an expression.  
Translate this into an expression.  
Clue Word: more \( 2 + n \)  
Clue Word: left \( 4 - n \)

Write the clues. Then write an expression using \( n \) for the unknown number.

**Explain what the variable represents.**

1. The temperature dropped 7 degrees and then went up 4 degrees.  
Clue Words: ____________________________  
   ____________________________  
   ____________________________  
   ____________________________

2. When the train stopped, 5 people boarded and 2 got off.  
Clue Words: ____________________________  
   ____________________________  
   ____________________________  
   ____________________________

3. Steven wrote 8 pages for homework. The dog ate some of them.  
Clue Words: ____________________________  
   ____________________________  
   ____________________________  
   ____________________________

Clue Words: ____________________________  
   ____________________________  
   ____________________________  
   ____________________________
Write Equations

An equation is a number sentence that uses the equal sign to show that two amounts are equal.

You can use variables to stand for numbers you do not know.

Peter had 10 books. After his birthday party, he had 16 books. How many books did he receive for his birthday?

books he has plus books received = total books

10 books + books received = total books

10 + n = 16

Write an equation with a variable. Explain what the variable represents.

1. Joseph had 7 paper cups. There were 22 students in the class. How many more cups did he need to serve punch to all his classmates?

   cups he had + cups he needed = total cups for punch

2. Mary Beth loves chocolate chip cookies. Her mother took a sheet of 12 out of the oven. Mary Beth ate some. Now there are 8 left. How many did she eat?

   total cookies − number eaten = number left

3. Jennifer had spent $32 for a new jacket. She had $12 left. How much did she have originally?

   original amount − amount spent = amount left

4. Monica had a collection of stickers. She bought 7 and had a total of 21. How many did she originally have?

   number in collection + number gained = total amount
Solve Equations

When you solve an equation, you find the value of the variable that makes the equation true.

In an equation, the amounts on both sides of the equal sign have the same value. It is like a balanced scale.

\[ n + 6 = 10 \]

To solve, ask, “How many counters would I need to add to the left side of the scale to make it balanced?” Use mental math to find the missing addend.

The solution equation will be

\[ n + 6 = 10. \text{ Think: what number plus 6 equals 10?} \]

\[ n = 4 \]

Check your solution. Replace \( n \) with 4.

\[ n + 6 = 10 \]
\[ 4 + 6 = 10 \]
\[ 10 = 10 \]

Use mental math to solve. Check your solution.

1. \( n + 5 = 15 \)
2. \( n - 6 = 6 \)
3. \( n - 10 = 20 \)

4. \( 15 + n = 22 \)
5. \( n - 8 = 12 \)
6. \( 25 - n = 22 \)

7. \( n + 10 - 6 = 7 \)
8. \( 22 - n + 7 = 18 \)
9. \( 14 - 8 + n = 13 \)
Use Addition Properties

You can use the properties of addition to help you solve problems.

The **Associative Property** states that you may group addends differently without changing the value of the sum.

\[
7 + (8 + 4) = (7 + 8) + 4 \\
7 + 12 = 15 + 4 \\
19 = 19
\]

The **Commutative Property** states that addends may be added in any order without changing the value of the sum.

\[
6 + 5 = 5 + 6 \\
11 = 11
\]

The **Zero Property** states that you may add zero to any number without changing the value of the number.

\[
5 + 0 = 5
\]

Name the addition property used in each equation.

1. \(223 + 0 = 223\)  
2. \((5 + 6) + 3 = 5 + (6 + 3)\)  
3. \(56.4 + 10 = 10 + 56.4\)

Find the value of \(n\). Identify the addition property used.

4. \(200 + n = 100 + 200\)  
5. \(78 + (5 + n) = (78 + 5) + 7\)

6. \(4 + n = 7 + 4\)  
7. \(0 + 88 = n\)

Algebra: Name the addition property used in each equation.

8. \(g + h = h + g\)  
9. \(p + (q + r) = (p + q) + r\)

10. \(w + 0 = w\)  
11. \(d + f = f + d\)
Problem Solving Skill

Use a Formula

To find the perimeter of a figure, you add the lengths of its sides. Remember that perimeter measures the distance around a figure.

You can use a formula to find the perimeter. Use a different letter for each side of the figure.

\[
P = a + b + c
\]

\[
P = 10 + 10 + 12
\]

\[
P = 32
\]

A triangle needs 3 letters.

Find the perimeter of the following figures.

1. \[
\begin{array}{c}
5 \\
8
\end{array}
\]

2. \[
\begin{array}{c}
5 \\
12
\end{array}
\]

3. \[
\begin{array}{c}
12 \\
15 \\
22
\end{array}
\]

4. \[
\begin{array}{c}
15 \\
20 \\
34
\end{array}
\]

Use a formula to solve.

5. Jeff wants to build a rectangular fence in his yard for his dog. The yard is 35 feet by 40 feet. How much fencing must Jeff buy?

6. Draw a square with each side measuring 8 units and find the perimeter.
Write and Evaluate Expressions

You can write and evaluate expressions to model different situations.

Ms. Hartwick has 6 rows of students in her classroom. She has the same number of students in each row.

To model how many students are in Ms. Hartwick’s class, you can write an expression.

\[
\text{6 rows times number of students in each row} \downarrow \downarrow \downarrow \\
\text{6} \times n
\]

If there are 5 students in each row, how many students are in Ms. Hartwick’s class altogether?

Replace the variable \( n \) in the expression with 5 to find how many students are in Ms. Hartwick’s class altogether.

\[
6 \times n \quad \text{Evaluate the expression if} \quad n = 5. \\
\downarrow \\
6 \times 5 \quad \text{Replace} \quad n \quad \text{with} \quad 5. \\
\downarrow \\
30
\]

So, there are 30 students in Ms. Hartwick’s class altogether.

Write an expression. If you use a variable, tell what it represents.

1. Beth runs 4 days a week. She runs the same number of miles each day.

2. Caitlin bought 12 boxes of canned dog food. Each box had 9 cans of dog food.

3. Marcus has 7 shelves of CDs. Each shelf holds the same number of CDs.

4. \( n \times 2 \) if \( n = 12 \) 

5. \( 9 \times n \) if \( n = 7 \) 

6. \( 22 + (n \times 3) \) if \( n = 6 \)
Order of Operations

When an expression has more than one operation, you evaluate it using the order of operations. The order of operations is a set of rules that tells you which operation to do first.

Evaluate $18 + (4 \times 6) \div 2$.

**Step 1** Operate inside parentheses.

$18 + (4 \times 6) \div 2$

$4 \times 6 = 24$

$18 + 24 \div 2$

$24 \div 2 = 12$

**Step 2** Multiply and divide from left to right.

$18 + 12$

**Step 3** Add and Subtract from left to right.

$18 + 12 = 30$

So, $18 + (4 \times 6) \div 2 = 30$

Complete to evaluate the expression.

1. $10 + (7 \times 4) - 8$
   
   $10 + ______ - 8$
   
   ______ - 8

2. $15 \div 5 \times 9 - 4$
   
   ______ \times 9 - 4
   
   ______ - 4

Evaluate the expression.

3. $14 - (5 + 2) \times 2$

   __________

4. $2 \times 8 + (16 \div 4)$

   __________

5. $5 \times 7 - 24 \div 8$

   __________

6. $4 + (55 \div 11) \times 6$

   __________

7. $29 - (6 \times 3) \div 2$

   __________

8. $(27 \div 9) \times 8 + 7$

   __________

9. $8 \times 6 - 7 \times 2$

   __________

10. $30 - (10 \div 10) + 13$

    __________

11. $6 \times 7 - 4 \times 5$

    __________

12. $19 - 7 \times (12 \div 6)$

    __________

13. $7 + 48 \div (7 + 5)$

    __________

14. $27 \div 3 - (1 \times 5)$

    __________
Functions

When one quantity depends on another quantity, the relationship between the quantities is called a function.

Paintbrushes cost $4 each. How much will 5 paintbrushes cost?

You can write an equation to represent the function.

Number of dollars \( d \) = \( 4 \times \) the number of paintbrushes \( p \)

\[
d = 4 \times p
\]

\[
d = 4 \times 5 = 20
\]

You can also use a function table to show the number of dollars different numbers of paintbrushes cost.

<table>
<thead>
<tr>
<th>paintbrushes, ( p )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>dollars, ( d )</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>

So, 5 paintbrushes will cost $20.

Complete the function table.

1. \( b = 9c \)

<table>
<thead>
<tr>
<th>( c )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. \( h = 6j + 4 \)

<table>
<thead>
<tr>
<th>( j )</th>
<th>8</th>
<th>6</th>
<th>4</th>
<th>2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. \( d = 3a - 2 \)

<table>
<thead>
<tr>
<th>( a )</th>
<th>12</th>
<th>10</th>
<th>8</th>
<th>6</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use the function. Find the output, \( y \) for each input, \( x \).

7. \( y = 8x - 7 \) for \( x = 3, 4, 5 \)

8. \( y = 100 - 4x \) for \( x = 5, 10, 20 \)

9. \( y = 6x + 15 \) for \( x = 6, 7, 8 \)

10. \( y = 49 - 3x \) for \( x = 8, 9, 10 \)
Problem Solving Strategy

Write an Equation

You can write an equation to help you solve a problem.

Felicity and Alex were in charge of parking cars in the small parking lot at the State Fair. The lot was filled with 72 cars in all by noon of the first day. The cars were organized into 9 equal rows of cars. How many cars were in each row?

Write an equation to find the number of cars parked in each row.

Think 9 times what number equals 72.

So, each row had 8 cars.

Write and solve an equation for each problem. Explain what the variable represents.

1. Jacob has to stack boxes in the grocer's storage room. The room is 96 inches high. Each box is 12 inches high. How many boxes can Jacob stack on top of each other?

2. The shelves that the grocer stacks the canned goods on are 30 inches high. The grocer stacked the cans 5 high. How tall is each can?

3. Chelsea has to line up 48 chairs in 6 equal rows. How many chairs should she put in each row?

4. Troy made a striped blanket for his bed. The blanket was 54 inches wide with 9 equal stripes. How wide was each stripe?
Use Multiplication Properties

You can use mental math and the *properties of multiplication* to solve problems.

<table>
<thead>
<tr>
<th>Property of Multiplication</th>
<th>Example</th>
<th>Explanation</th>
</tr>
</thead>
</table>
| Commutative Property        | $4 \times 2 = n \times 4$  
                             | $4 \times 2 = 2 \times 4$  
                             | $n = 2$                                                                 | You can multiply numbers in any order. The product is always the same. |
| Associative Property        | $(3 \times n) \times 5 = 3 \times (4 \times 5)$  
                             | $(3 \times 4) \times 5 = 3 \times (4 \times 5)$  
                             | $n = 4$                                                                 | You can group factors differently. The product is always the same. |
| Property of One             | $n \times 1 = 5$  
                             | $5 \times 1 = 5$  
                             | $n = 5$                                                                 | When one of the factors is 1, the product equals the other number. |
| Zero Property               | $4 \times n = 0$  
                             | $4 \times 0 = 0$  
                             | $n = 0$                                                                 | When one factor is 0, the product is 0. |

Solve the equation. Identify the property used.

1. $n \times 3 = 0$
2. $n \times 3 = 3 \times 2$
3. $4 \times (2 \times 5) = (n \times 2) \times 5$

4. $1 \times n = 8$
5. $(n \times 3) \times 2 = 5 \times (3 \times 2)$
6. $6 \times 7 = 7 \times n$

7. $(7 \times 3) \times n = 7 \times (3 \times 2)$
8. $8 \times 2 = n \times 8$
9. $3 \times n = 3$
The Distributive Property

You can use the **Distributive Property** to break apart numbers to make them easier to multiply.

To find $20 \times 13$, you can break apart 13.

$$20 \times 13 = 20 \times (10 + 3) \quad \leftarrow \text{Break apart.}$$

$$= (20 \times 10) + (20 \times 3) \quad \leftarrow \text{Multiply.}$$

$$= (200) + (60) \quad \leftarrow \text{Add.}$$

$$= 260$$

Use the Distributive Property to restate each expression. Find the product.

1. $20 \times 12$

   Break apart. $20 \times (\_ + \_)$

   Multiply. $20 \times \_ = \_$$

   $20 \times \_ = \_

   Add. $200 + \_ = \_$$

2. $20 \times 18$

   Break apart. $20 \times (\_ + \_)$

   Multiply. $20 \times \_ = \_$$

   $20 \times \_ = \_

   Add. $200 + \_ = \_$$

3. $30 \times 16$

   Break apart. $30 \times (\_ + \_)$

   Multiply. $30 \times \_ = \_$$

   $30 \times \_ = \_

   Add. $\_ + \_ = \_$$

4. $12 \times 45$

   Break apart. $12 \times (\_ + \_)$

   Multiply. $12 \times \_ = \_$$

   $12 \times \_ = \_

   Add. $\_ + \_ = \_$$

5. $30 \times 26$

   Break apart. $30 \times (\_ + \_)$

   Multiply. $30 \times \_ = \_$$

   $30 \times \_ = \_

   Add. $\_ + \_ = \_$$

6. $25 \times 17$

   Break apart. $25 \times (\_ + \_)$

   Multiply. $25 \times \_ = \_$$

   $25 \times \_ = \_

   Add. $\_ + \_ = \_$$
Collect and Organize Data

The tally table shows how many fifth grade students rode the bus during the first four weeks of school. How can you find the total number of students in the fifth grade that rode a bus to school?

The information in the tally table can be made easier to read and understand by using a frequency table. The frequency for each week tells how many fifth grade students rode a bus that week. The cumulative frequency column shows a running total of the number of students who rode a bus.

**Step 1** Count the tally marks for each week. Place the total for each week in the column labeled Frequency on the frequency table.

**Step 2** For each new line of data, write the sum of the frequencies in the Cumulative Frequency column. The last number in the Cumulative Frequency column will tell you the total number of fifth graders that rode a bus.

How many fifth graders rode a bus? _____

The range is the difference between the greatest and the least numbers in a set of data. Greatest Number – Least Number = Range

Use the frequency table to find the range of the number of fifth graders who rode a bus. Show your work. _______________

Suppose 2 more fifth graders rode a bus in Week 3. In addition, 7 new fifth graders enrolled in school. 4 of the new students are walkers and 3 rode a bus in Week 2. Use this information to complete a new frequency table. What is the new total number of fifth graders riding a bus? What is the new range?

______________
Find the Mean

Tom has taken three tests. He wants to know his average score for the three tests. The type of average Tom is looking for is called the **mean**.

**Step 1**
Add the three test scores together.

\[
80 + 70 + 90 = 240
\]

**Step 2**
Divide the sum by the number of tests.

\[
240 \div 3 = 80
\]

So, Tom’s mean test score is 80.

---

Write an addition sentence for the sum of each set of numbers.

1. 3, 5, 4, 1, 7

2. 20, 15, 10

3. 22, 26, 28, 32

Write how many numbers are listed in each set of numbers.

4. 3, 5, 4, 1, 7

5. 20, 15, 10

6. 22, 26, 28, 32

Write a division sentence to find the mean for each set of numbers.

7. 3, 5, 4, 1, 7

8. 20, 15, 10

9. 22, 26, 28, 32

10. One month later, Tom took 5 more tests. His scores were 80, 70, 90, 90, and 100. What is the mean of these test scores? Show your work.

---

Find the mean for each set of data.

11. 9, 11, 13, 13, 9

12. 33, 28, 35, 33, 26

13. 105, 112, 133, 118, 102
Find the Median and Mode

Sam takes tests to see how many words he can type in a minute. The data in the table show his first 7 tests.

<table>
<thead>
<tr>
<th>Number of Words Typed in a Minute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
</tr>
<tr>
<td>Score</td>
</tr>
</tbody>
</table>

You can find Sam’s median score and the mode of the data.

**Step 1**
List the scores from least to greatest.
14, 16, 16, 18, 20, 22, 34

**Step 2**
To find the median score, cross off a number from each end until there is only one number left in the middle.
14, 16, 16, 18, 20, 22, 34

The number 18 is the median score.

**Step 3**
Find the score that occurred most often.
Sam scored 16 twice.

The number 16 is the mode.

Sometimes there is more than one mode or no mode.

Arrange the numbers from least to greatest. Circle the median number.

1. 13, 12, 11, 11, 9, 8, 16, 17, 19
2. 24, 32, 28, 45, 19, 23, 16, 51, 32
3. 103, 98, 105, 101, 99

Arrange the numbers from least to greatest. Find the median and the mode.

4. 9, 7, 5, 11, 11
5. 14, 12, 12
6. 3, 7, 2, 9, 6, 5, 3, 1, 3

**median:** 
**mode:** 

**median:** 
**mode:** 

**median:** 
**mode:**
Problem Solving Strategy

Make a Graph

Mr. Schwartz recorded the number of newspapers he sold in his store every day of the week for two weeks. Newspapers sales were 60, 65, 66, 71, 71, 72, 74, 75, 76, 77, 79, 80, 81, and 83. Is the number sold usually in the 60's, 70's, or 80's?

You can make a stem-and-leaf plot to organize the data by place value.

Make a column of the tens digits of the data, listing them in order from least to greatest. These are the stems.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

Beside each tens digit, record the ones digits of the data, in order from least to greatest. These are the leaves.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0 5 6</td>
</tr>
<tr>
<td>7</td>
<td>1 1 2 4 5 6 7 9</td>
</tr>
<tr>
<td>8</td>
<td>0 1 3</td>
</tr>
</tbody>
</table>

The stem-and-leaf plot shows the greatest number of leaves are on the 7 stem. So, the number of newspapers sold is usually in the 70's.

Make a graph to solve.

1. Lynnette's golf scores are 72, 74, 74, 78, 80, 82, 83, 87, 88, and 91. Does she usually score in the 70's, 80's, or 90's?

2. The coach of the Tigers recorded the number of parents that attended each home baseball game. Parents' attendance was 16, 17, 23, 24, 29, 30, 33, 36, 36, and 38. Is parents' attendance usually in the 10's, 20's, or 30's?
Analyze Graphs

Graphs help you to draw conclusions, answer questions, and make predictions about the data. Study the following graphs to answer the questions.

1. A **bar graph** is useful when comparing data by groups.
   Which student read the most books? the least?

2. **Line graphs** are helpful to see how data changes over a period of time.
   What happened to the temperature as the week passed?

3. A **circle graph** shows how parts of data relate to each other and to the whole.
   About one half of the animals in the pet store are what type of animal?

4. A **pictograph** displays countable data with symbols or pictures. Pictographs have a key to show how many each picture represents.
   How many books does Mr. Williams have in his class? ________________
Choose a Reasonable Scale

Henry kept track of how much mail his family received in one week.

He put the data in a table.

He wants to put the data in a line graph. He must select a scale. A **scale** is the series of numbers placed at fixed distances. The difference between one number and the next on the scale is called the **interval**.

The scale must include the numbers 2 through 10. It must include a number less than the least data and a number greater than the greatest data. Look at four ways Henry can display the mail data.

Henry selects a scale with intervals of 2.

From the box, choose the most reasonable interval for each set of data. List the numbers needed in the scale.

1. 5, 15, 20, 25, 10, 18
   
2. 50, 125, 100, 150, 100, 20
   
3. 8, 12, 10, 20, 10, 30
   
4. 20, 101, 40, 59, 115

<table>
<thead>
<tr>
<th>Interval</th>
<th>a. By 25’s</th>
<th>b. By 20’s</th>
<th>c. By 10’s</th>
<th>d. By 5’s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Problem Solving Strategy:

Make a Graph

The school population has changed over the last five years. Sid wants to use this data to predict next year’s school population. He organized the data into a table.

Then he planned how to display the data using a bar graph. The interval skipped from 0 to 450, so Sid used a zigzag line to show a break in the scale.

He finds the range is 170.

He chooses the interval of 50.

Using the graph, Sid predicts that next year’s population will be about 650 students.

Make a graph to solve.

1. Mr. Struther surveyed some students to find ideas for a field trip. He organized the data into a table. What graph or plot should he use to display the data? Make a graph or plot.

2. Attendance at the zoo was organized into a table. What graph or plot would best display the data? Make a graph or plot.

3. A baseball team kept track of the number of parents at the baseball games. The team organized the data into a table. What graph or plot would best display the data? Make a graph or plot.
Graph Ordered Pairs

Points on a coordinate grid can be given a unique name in the same way each house on a street has a unique number. Houses on a street follow an order so people can tell them apart and points also follow an order.

The order of the numbers in an ordered pair is always expressed the same way. The first number in an ordered pair tells how far to move horizontally from the origin. The second number tells how far to move vertically.

Name the ordered pair for each point.

1. \( E \) ________ 2. \( H \) ________
3. \( O \) ________ 4. \( C \) ________
5. \( A \) ________ 6. \( D \) ________
7. \( N \) ________ 8. \( I \) ________
9. \( W \) ________ 10. \( L \) ________

Graph and label the following points on a coordinate grid.

11. \( M \ \text{(5, 7)} \) 12. \( N \ \text{(0, 5)} \) 13. \( P \ \text{(3, 4)} \) 14. \( R \ \text{(1, 0)} \)
15. \( S \ \text{(6, 2)} \) 16. \( A \ \text{(2, 5)} \) 17. \( V \ \text{(4, 1)} \) 18. \( G \ \text{(3, 7)} \)
19. \( B \ \text{(6, 0)} \) 20. \( H \ \text{(2, 6)} \) 21. \( T \ \text{(1, 7)} \) 22. \( Y \ \text{(6, 3)} \)
Make Line Graphs

The table shows how much money the XYZ Toy Company made for the last five years. The company wants to display the data in a line graph.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales in Millions ($)</td>
<td>120</td>
<td>60</td>
<td>80</td>
<td>140</td>
<td>160</td>
</tr>
</tbody>
</table>

The greatest number in the table is 160 million. The least number is 60 million. The difference between the greatest and the least number in the set of data is the range.

\[160,000,000 - 60,000,000 = 100,000,000\]

There is a break in the scale from 0 to 60 and the interval is 20.

The vertical axis is labeled with the amounts of money; the horizontal axis is labeled with the years.

For each set of data, write a subtraction sentence to find the range.

1. 12, 18, 9, 13, 7
2. 20, 100, 40, 60, 35
3. 1, 9, 6, 5, 3

Make a line graph for each set of data.

4. On-Line Hours Used

<table>
<thead>
<tr>
<th>Month</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Jan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours Used</td>
<td>30</td>
<td>60</td>
<td>100</td>
<td>125</td>
</tr>
</tbody>
</table>

5. Jelly Bean Sales

<table>
<thead>
<tr>
<th>Days</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boxes Sold</td>
<td>100</td>
<td>50</td>
<td>150</td>
<td>250</td>
<td>200</td>
</tr>
</tbody>
</table>
Histograms

Histograms are a type of bar graph. The bars in a histogram are related and follow an order. They show the number of times the data occur within intervals.

The bars in a bar graph are not related to one another. To decide whether to make a histogram or bar graph, you need to decide whether the data fall within intervals.

Every 30 minutes, the popcorn popper records the number of boxes sold in the movie theater. Here is the information.

<table>
<thead>
<tr>
<th>Time</th>
<th>5:00</th>
<th>5:30</th>
<th>6:00</th>
<th>6:30</th>
<th>7:00</th>
<th>7:30</th>
<th>8:00</th>
<th>8:30</th>
<th>9:00</th>
<th>9:30</th>
<th>10:00</th>
<th>10:30</th>
<th>11:00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>28</td>
<td>26</td>
<td>33</td>
<td>45</td>
<td>56</td>
<td>86</td>
<td>85</td>
<td>57</td>
<td>25</td>
<td>35</td>
<td>48</td>
<td>32</td>
<td>21</td>
</tr>
</tbody>
</table>

Find the range for the set of data.

What interval could you use to make the histogram? Select an interval to divide the data equally.

Make a frequency table with the intervals and record the number of boxes of popcorn sold during these time periods.

Use the frequency table to make the histogram. Label the scale for the number of boxes sold and title the graph. Graph the frequency for each interval.

Remember that the bars in a histogram are side-by-side.

Decide which graph would better represent the data below, a bar graph or histogram. Then make the graph.

<table>
<thead>
<tr>
<th>MONEY SPENT ON LUNCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of Money</td>
</tr>
<tr>
<td>$1.25</td>
</tr>
<tr>
<td>$1.75</td>
</tr>
<tr>
<td>$2.00</td>
</tr>
<tr>
<td>$2.50</td>
</tr>
<tr>
<td>$2.75</td>
</tr>
<tr>
<td>$3.00</td>
</tr>
</tbody>
</table>
Choose the Appropriate Graph

To display data, it is important to select the most appropriate graph or plot.

Two different graphs and two different plots for displaying data are shown.

### Line Plot

A line plot is used to record data as they are collected.

\[
\begin{array}{c}
\text{score} \\
75 \\
87 \\
92 \\
95 \\
100
\end{array}
\]

### Bar Graph

A bar graph is used to compare facts about groups.

### Stem-and-Leaf Plot

A stem-and-leaf plot is used to organize data by place value.

<table>
<thead>
<tr>
<th>stem</th>
<th>leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>2 5</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

Key: \(7|5\) = a score of 75

### Line Graph

A line graph shows change over time.

Michael chose the line graph because it shows his test scores over time.

Write the best type of graph or plot for the data.

1. Compare the population of 6 major cities.
2. Record the letter grades (A–F) for a class of 30 students.
3. Display the daily growth of a sunflower in inches.
4. Record the temperature every hour for 24 hours.
Estimation: Patterns in Multiples

You can round numbers and use basic facts to estimate products. Count the number of zeros in your rounded numbers. They will appear to the right of your basic fact in your estimate.

For 2-digit numbers:
If the ones digit is 0–4, round down.
If the ones digit is 5–9, round up.

For example: Round 41–44 to 40. Round 45–49 to 50.

\[
\begin{align*}
45 & \rightarrow 50 > 2 \text{ zeros} \\
\times 42 & \rightarrow \times 40 \\
2,000 & \\
\end{align*}
\]

For 3-digit numbers:
If the tens digit is 0–4, round down.
If the tens digit is 5–9, round up.

For example: Round 700–749 to 700. Round 750–799 to 800.

\[
\begin{align*}
749 & \rightarrow 700 > 3 \text{ zeros} \\
\times 44 & \rightarrow \times 40 \\
28,000 & \\
\end{align*}
\]

Round each factor and estimate the product.

1. \(141 \rightarrow \times 36 \rightarrow \times \ldots\)
2. \(157 \rightarrow \times 57 \rightarrow \times \ldots\)
3. \(125 \rightarrow \times 25 \rightarrow \times \ldots\)

4. \(160 \rightarrow \times 41 \rightarrow \times \ldots\)
5. \(187 \rightarrow \times 72 \rightarrow \times \ldots\)
6. \(236 \rightarrow \times 45 \rightarrow \times \ldots\)

7. \(349 \rightarrow \times 74 \rightarrow \times \ldots\)
8. \(456 \rightarrow \times 56 \rightarrow \times \ldots\)
9. \(568 \rightarrow \times 27 \rightarrow \times \ldots\)

10. \(638 \rightarrow \times 16 \rightarrow \times \ldots\)
11. \(774 \rightarrow \times 55 \rightarrow \times \ldots\)
12. \(836 \rightarrow \times 43 \rightarrow \times \ldots\)

13. \(719 \rightarrow \times 85 \rightarrow \times \ldots\)
14. \(468 \rightarrow \times 68 \rightarrow \times \ldots\)
15. \(229 \rightarrow \times 54 \rightarrow \times \ldots\)
Multiply by 1-Digit Numbers

Multiply the ones. Multiply the tens. Multiply the hundreds.

\[
\begin{array}{c}
143 \\
\times 3 \\
\hline
9
\end{array}
\quad \quad \quad
\begin{array}{c}
143 \\
\times 3 \\
\hline
29
\end{array}
\quad \quad \quad
\begin{array}{c}
143 \\
\times 3 \\
\hline
429
\end{array}
\]

Sometimes you need to regroup.

Step 1 Multiply the ones. \(3 \times 3\) ones = 9 ones

\[
3 \quad \times \quad 3 \\
\hline
9
\]

Step 2 Multiply the tens. \(3 \times 4\) tens = 12 tens
Write the 2. Regroup the 10 tens as 1 hundred.

\[
1 \quad 43 \\
\times \quad 3 \\
\hline
29
\]

Step 3 Multiply the hundreds. \(3 \times 1\) hundred = 3 hundreds
Now add the regrouped hundred.

\[
1 \quad 43 \\
\times \quad 3 \\
\hline
429
\]

So, \(3 \times 143 = 429\).

Tell which place-value positions must be regrouped. Find the product.

1. \[
\begin{array}{c}
451 \\
\times 2 \\
\hline
\end{array}
\]

2. \[
\begin{array}{c}
328 \\
\times 3 \\
\hline
\end{array}
\]

3. \[
\begin{array}{c}
715 \\
\times 5 \\
\hline
\end{array}
\]

4. \[
\begin{array}{c}
1,458 \\
\times 6 \\
\hline
\end{array}
\]

5. \[
\begin{array}{c}
2,473 \\
\times 2 \\
\hline
\end{array}
\]

6. \[
\begin{array}{c}
6,925 \\
\times 4 \\
\hline
\end{array}
\]

7. \[
\begin{array}{c}
3,562 \\
\times 7 \\
\hline
\end{array}
\]

8. \[
\begin{array}{c}
20,317 \\
\times 4 \\
\hline
\end{array}
\]

9. \[
\begin{array}{c}
13,234 \\
\times 3 \\
\hline
\end{array}
\]
Multiply by 2-Digit Numbers

You can multiply by two-digit numbers by breaking apart one of the factors.

To find $21 \times 14$, you can break apart 14 into 1 ten 4 ones.

**Step 1** Multiply by the ones.

$$
\begin{array}{c}
21 \\
\times 4 \\
\hline \\
84
\end{array}
$$

**Step 2** Multiply by the tens.

$$
\begin{array}{c}
21 \\
\times 10 \\
\hline \\
210
\end{array}
$$

**Step 3** Add the products.

$$
\begin{array}{c}
21 \\
\times 14 \\
\hline \\
84 \\ +210 \\
\hline \\
294
\end{array}
$$

So, $21 \times 14 = 294$.

Complete to find the product.

1. $13 \times 12$

$$
\begin{array}{c}
13 \\
\times 12 \\
\hline \\
+____ \\
\hline
\end{array}
$$

2. $22 \times 15$

$$
\begin{array}{c}
22 \\
\times 15 \\
\hline \\
+____ \\
\hline
\end{array}
$$

3. $30 \times 17$

$$
\begin{array}{c}
30 \\
\times 17 \\
\hline \\
+____ \\
\hline
\end{array}
$$

4. $28 \times 14$

$$
\begin{array}{c}
28 \\
\times 14 \\
\hline \\
+____ \\
\hline
\end{array}
$$

5. $40 \times 19$

$$
\begin{array}{c}
40 \\
\times 19 \\
\hline \\
+____ \\
\hline
\end{array}
$$

6. $45 \times 15$

$$
\begin{array}{c}
45 \\
\times 15 \\
\hline \\
+____ \\
\hline
\end{array}
$$

7. $37 \times 15$

$$
\begin{array}{c}
37 \\
\times 15 \\
\hline \\
+____ \\
\hline
\end{array}
$$

8. $28 \times 16$

$$
\begin{array}{c}
28 \\
\times 16 \\
\hline \\
+____ \\
\hline
\end{array}
$$
Choose a Method

You can multiply three-digit numbers by breaking apart one of the factors.

To find $312 \times 143$, break apart 143 into 1 hundred 4 tens 3 ones.

**Step 1**
Multiply by the ones.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>312</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\times$ 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>936</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 2**
Multiply by the tens.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>312</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\times$ 40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12,480</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 3**
Multiply by the hundreds.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>312</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\times$ 100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31,200</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 4**
Add the products.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>312</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\times$ 143</td>
<td></td>
<td></td>
</tr>
<tr>
<td>936 $\leftarrow$ 3 $\times$ 312</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12,480 $\leftarrow$ 40 $\times$ 312</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31,200 $\leftarrow$ 100 $\times$ 312</td>
<td></td>
<td></td>
</tr>
<tr>
<td>44,616</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

So, $312 \times 143 = 44,616$.

Complete to find the product.

1. $423 \times 146$

   $\leftarrow$ _____ $\times$ _____

   $\leftarrow$ _____ $\times$ _____

   + _____ $\leftarrow$ _____ $\times$ _____

2. $231 \times 123$

   $\leftarrow$ _____ $\times$ _____

   $\leftarrow$ _____ $\times$ _____

   + _____ $\leftarrow$ _____ $\times$ _____

3. $354 \times 246$

   $\leftarrow$ _____ $\times$ _____

   $\leftarrow$ _____ $\times$ _____

   + _____ $\leftarrow$ _____ $\times$ _____

4. $438 \times 253$

   $\leftarrow$ _____ $\times$ _____

   $\leftarrow$ _____ $\times$ _____

   + _____ $\leftarrow$ _____ $\times$ _____

5. $672 \times 334$

   $\leftarrow$ _____ $\times$ _____

   $\leftarrow$ _____ $\times$ _____

   + _____ $\leftarrow$ _____ $\times$ _____

6. $596 \times 254$

   $\leftarrow$ _____ $\times$ _____

   $\leftarrow$ _____ $\times$ _____

   + _____ $\leftarrow$ _____ $\times$ _____
Problem Solving Skill

Evaluate Answers for Reasonableness

You can use estimation to check if an answer is reasonable. Use your knowledge of patterns in multiples to help you with large numbers.

At the garden center, there were 174 rows of flowers. Each row contained 86 flowers. Estimate first. Then solve the problem and compare it to your estimate to see if it is reasonable.

Estimate

\[
\begin{array}{c}
200 \\
\times 90 \\
\hline
18,000 \\
\end{array}
\quad \begin{array}{c}
174 \\
\times 86 \\
\hline
1,044 \\
+13,920 \\
\hline
14,964 \\
\end{array}
\]

Your estimate was more than your exact amount because you used greater numbers.

How would your estimate compare to your exact answer if you rounded both factors down?

Choose the most reasonable answer without solving.

1. Joel’s dad sold each of his paintings at an art show for $750. He sold 26 at the show. About how much money did he get?

   A $7,000       C $20,000
   B $12,000      D $60,000

2. Joel’s dad pays $157 for the materials to create each of his paintings. About how much does it cost him to create the 26 paintings he sold?

   F $600       H $5,000
   G $1,500     J $60,000

3. A small airport has 21,795 passengers each year. About how many passengers will they have altogether in 8 years?

   A 2,000       C 200,000
   B 20,000      D 2,000,000

4. The average length of important rivers in the world is 2,142 miles. If we measured 18 of these rivers, about how many miles would we measure?

   F 4,000 miles   H 40,000 miles
   G 20,000 miles  J 400,000 miles
Multiply Decimals and Whole Numbers

To multiply a whole number and a decimal, modeling with money can be helpful. To multiply $3 \times 0.2$, follow these steps.

**Step 1**
Write 0.2 as 0.20. You can add a zero at the end of a decimal without changing the value. Draw that amount of money.

The 2 dimes equal $0.20. You could also draw 4 nickels or 20 pennies.

**Step 2**
Draw 3 groups of coins of $0.20.

Count the total amount.

$0.20 + $0.20 + $0.20 = $0.60

So, $3 \times 0.2 = 0.60$, or 0.6.

Draw the coins that equal the decimal amount. Use the fewest coins possible.

1. 0.28  
2. 0.30  
3. 0.16

4. 0.52  
5. 0.80  
6. 0.24

Make a money model to find each product.

7. $4 \times 0.15 = \underline{_______}$
8. $3 \times 0.1 = \underline{_______}$
9. $2 \times 0.21 = \underline{_______}$

10. $4 \times 0.01 = \underline{_______}$
11. $3 \times 0.06 = \underline{_______}$
12. $2 \times 0.78 = \underline{_______}$

13. $3 \times 0.32 = \underline{_______}$
14. $4 \times 0.12 = \underline{_______}$
15. $2 \times 0.53 = \underline{_______}$
Algebra: Patterns in Decimal Factors and Products

You can use patterns to place the decimal point in a product.

The number of decimal places in the factors equals the number of decimal places in the product.

Complete the tables.

1. $2 \times 3 = \quad 2 \times 0.3 = \quad 2 \times 0.03 =$

2. $2 \times 4 = \quad 2 \times 0.4 = \quad 2 \times 0.04 =$

3. $3 \times 3 = \quad 3 \times 0.3 = \quad 3 \times 0.03 =$

4. $3 \times 5 = \quad 3 \times 0.5 = \quad 3 \times 0.05 =$

5. $3 \times 6 = \quad 3 \times 0.6 = \quad 3 \times 0.06 =$

6. $3 \times 7 = \quad 3 \times 0.7 = \quad 3 \times 0.07 =$

7. $2 \times 8 = \quad 2 \times 0.8 = \quad 2 \times 0.08 =$

8. $4 \times 5 = \quad 4 \times 0.5 = \quad 4 \times 0.05 =$

9. $6 \times 7 = \quad 6 \times 0.7 = \quad 6 \times 0.07 =$

10. $15 \times 1 = \quad 15 \times 0.1 = \quad 15 \times 0.01 =$

11. $28 \times 1 = \quad 28 \times 0.1 = \quad 28 \times 0.01 =$

12. $32 \times 1 = \quad 32 \times 0.1 = \quad 32 \times 0.01 =$
Model Decimal Multiplication

To multiply $0.3 \times 0.2$, a 10-by-10 model will help.

**Step 1:** Draw diagonal lines through the bottom 3 rows.

**Step 2:** Draw diagonal lines through 2 columns.

**Step 3:** The overlapping squares that have an x in them show the product of $0.3 \times 0.2$.

The 3 rows represent 0.3.

The 2 columns represent 0.2.

The 6 squares with x's represent 0.06.

The product of $0.3 \times 0.2$ is 0.06.

Write a number sentence for each drawing.

1. 2. 3. 4.

Make a model for each and find the product.

5. $0.1 \times 0.5 = \underline{_{\_\_\_\_}}$

6. $0.2 \times 0.8 = \underline{_{\_\_\_\_}}$

7. $0.5 \times 0.9 = \underline{_{\_\_\_\_}}$

8. $0.7 \times 0.5 = \underline{_{\_\_\_\_}}$
Place the Decimal Point

How many decimal places are in the product of 0.21 and 0.03?

Step 1
Add the number of decimal places from each factor.

\[ 0.21 \times 0.03 = ? \]

2 places + 2 places = 4 places

\[ \begin{array}{c}
0.21 \\
\times \ 
0.03 \\
\hline
0. \_ \_ \_ \_ \\
\end{array} \]

Step 2
Multiply the numbers just like whole numbers. To have 4 decimal places, you have to add 2 zeros before the 63.

\[ \begin{array}{c}
0.21 \\
\times \ 
0.03 \\
\hline
0.063 \\
\end{array} \]

Write how many decimal places are in each number.

1. 0.105
2. 0.0006
3. 0.008

Write how many decimal places are in each product. Then write the product. The first one has been done for you.

4. 0.3 \times 0.5

= 0.15

5. 0.6 \times 0.03

= 0.018

6. 0.002 \times 0.8

= 0.0016

Find each product.

10. 0.5 \times 0.03 = ______

11. 0.06 \times 1.8 = ______

12. 7 \times 0.08 = ______
Zeros in the Product

Be careful when multiplying by decimals to include all of the decimal places in the product.

Example: Find $0.013 \times 0.6$.

1. Find 0.03 $\times$ 0.4.
2. Find 0.047 $\times$ 0.07.
3. Find 0.0732 $\times$ 0.8.
4. Find 0.054 $\times$ 0.007.
5. Find 0.0942 $\times$ 0.7.
Problem Solving Skill

Make Decisions

We make decisions every day. There are often many things to consider. Use the questions below to guide you through making decisions.

Your neighbors have invited you to go with them on Saturday. Julia's family is going to the museum and to a movie. Karl's family is going on a bakery tour and to a football game. You must decide which invitation to accept.

1. If the museum visit will cost $3.00 and the movie will cost $4.75, how much will the trip with Julia's family cost?

2. If a football ticket costs $14.50 and the bakery tour is free, how much will the trip with Karl's family cost?

3. If you had to make your decision based on total cost, which trip would you choose? Why?

4. Julia's family will start their trip at 8:30 A.M. Breakfast will take 45 minutes. They plan to stay at the museum for 2 hours. Lunch will take 45 minutes, and the movie will last 2 hours and 30 minutes. When will the trip with Julia's family end?

5. The bakery tour will take 1 hour and 30 minutes. Lunch will take 30 minutes. The football game will take 3 hours and 30 minutes, and dinner with Karl's family will take 1 hour. If this trip starts at 11:00 A.M., when will it end?

6. If you had to make your decision based on the total time of the trip, the start time, or the end time of the trip, which invitation would you accept? Why?

7. If you had to make your decision based on the activities you like better, which invitation would you accept? Why?


Estimate Quotients

Compatible numbers are numbers that are easy to compute mentally. One compatible number divides evenly into the other. Think of number factors to help you find compatible numbers.

What is $\frac{8}{554}$?

**Step 1**

Think: What are the multiples of 8?

<table>
<thead>
<tr>
<th>8</th>
<th>16</th>
<th>24</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>48</td>
<td>56</td>
<td>64</td>
</tr>
</tbody>
</table>

Which multiple is closest to 55?

56 is close to 55.

8 and 560 are compatible numbers.

**Step 2**

Divide.

$560 \div 8 = 70$

A good estimate for $554 \div 8$ is 70.

Follow the steps above to estimate each quotient.

1. $3 \div 252$
2. $6 \div 546$
3. $4 \div 154$
4. $9 \div 192$

5. $7 \div 129$
6. $4 \div 265$
7. $8 \div 344$
8. $5 \div 480$

9. $2 \div 497$
10. $3 \div 287$
11. $5 \div 6,558$
12. $6 \div 5,097$
Divide 3-Digit Dividends

Bryan has 522 coins. He divides them among 3 jars. How many coins are in each jar?

Divide. 522 ÷ 3 = n

Step 1
Since 5 hundreds can be divided by 3, the first digit is in the hundreds place.

Divide. 3|522
   Multiply. 3 × 1
   Subtract. 5 − 3
   Compare. 2 < 3

Step 2
Bring down the tens. Divide. 3|22
   Multiply. 3 × 7
   Subtract. 22 − 21
   Compare. 1 < 3

Step 3
Bring down the ones. Divide. 3|12
   Multiply. 3 × 4
   Subtract. 12 − 12
   Compare. 0 < 3

Since n = 174, each jar contains 174 coins.

Follow the steps above to find each quotient.

1. 3|928
2. 7|149
3. 5|845
4. 4|892

5. 6|399
6. 3|873
7. 9|765
8. 5|934
Zeros in Division

There are 618 pencils in the supply room. They are to be divided evenly among 6 classes. How many pencils will each class receive?

You will use division to find the answer. $618 \div 6 = n$

**Step 1**

Since 6 hundreds can be divided by 6, the first digit will be in the hundreds place. Divide.

$$\begin{array}{c|c|c|c|c}
\text{Divisor} & \text{Dividend} & \text{Quotient} & \text{Remainder} \\
6 & 618 & 1 & 0 \\
\end{array}$$

 Multiply.

$6 \times 1 = 6$

 Subtract.

$6 - 6 = 0$

 Compare.

$0 < 6$

**Step 2**

Bring down the tens. Divide the 1 ten. Since 6 > 1, write 0 in the quotient.

$$\begin{array}{c|c|c|c|c}
\text{Divisor} & \text{Dividend} & \text{Quotient} & \text{Remainder} \\
6 & 618 & 0 & 1 \\
\end{array}$$

 Multiply.

$6 \times 0 = 0$

 Subtract.

$1 - 0 = 1$

 Compare.

$1 < 6$

**Step 3**

Bring down the ones. Divide.

$$\begin{array}{c|c|c|c|c}
\text{Divisor} & \text{Dividend} & \text{Quotient} & \text{Remainder} \\
6 & 618 & 103 & 18 \\
\end{array}$$

 Multiply.

$6 \times 3 = 18$

 Subtract.

$18 - 18 = 0$

 Compare.

$0 < 6$

So, each class will receive 103 pencils.

Follow the steps above to find each quotient.

1. $3\overline{927}$
2. $8\overline{872}$
3. $5\overline{542}$
4. $6\overline{608}$

5. $3\overline{624}$
6. $2\overline{807}$
7. $4\overline{826}$
8. $7\overline{843}$
Choose a Method

Divide 42,574 by 7.

**Divide** 7 into 42 to get 6. Multiply 6 by 7 to get 42. Subtract 42 from 42 to get 0. Bring down the 5 to get 05.

**Divide** 7 into 5 to get 0; Multiply 0 by 7 to get 0. Subtract 0 from 05 to get 5. Bring down the 7 to get 57.

**Divide** 7 into 57 to get 8. Multiply 8 by 7 to get 56. Subtract 56 from 57 to get 1. Bring down the 4 to get 14.

**Divide** 7 into 14 to get 2. Multiply 2 by 7 to get 14. Subtract 14 from 14 to get 0.

Answer = 6,082

Follow the steps above to find each quotient.

1. 9)45,035  
2. 5)9,085  
3. 4)16,087  
4. 5)70,861  
5. 6)856,412  
6. 5)18,005  
7. 4)200,088  
8. 5)7,555  
9. 5)654,321  
10. 5)21,076  
11. 3)356,789  
12. 3)67,530  
13. 6)3,791  
14. 7)4,326  
15. 8)1,999  
16. 3)645,123
Algebra: Expressions and Equations

An **expression** combines numbers or variables with operations.

**Problem:** twenty-four divided by a number  
**Expression:** \(24 \div n\)

The **value** of the expression depends on the number, \(n\). If the number is 2, the value is 12. If the number is 8, the value is 3.

An **equation** is a number sentence that uses an equal sign to show that two amounts are equal.

**Problem:** Twenty-four divided by a number is six.  
**Equation:** \(24 \div n = 6\)

To solve the equation, think: 24 divided by what number equals 6?
You can predict and test to solve.

Predict: 3  Test: 24 ÷ 3 = 8; too high  
Predict: 4  Test: 24 ÷ 4 = 6; correct

Evaluate the expression for \(n\).

1. \(48 \div n\)  
   \(n = 2, 6, 12\)
2. \(n \div 10\)  
   \(n = 100, 60, 70\)
3. \(n \div 12\)  
   \(n = 12, 36, 72\)
4. \(18 \div n\)  
   \(n = 2, 6, 18\)

Determine which value is a solution for the given equation.

5. \(49 \div n = 7\)  
   \(n = 5, 6, \text{ or } 7\)
6. \(195 \div n = 65\)  
   \(n = 2, 3, \text{ or } 4\)
7. \(n \div 6 = 8\)  
   \(n = 36, 42, \text{ or } 48\)
8. \(n \div 5 = 50\)  
   \(n = 200, 250, \text{ or } 300\)

9. \(350 \div n = 50\)  
   \(n = 5, 6, \text{ or } 7\)
10. \(200 \div n = 5\)  
    \(n = 30, 40, \text{ or } 50\)
11. \(n \div 6 = 12\)  
    \(n = 72, 76, \text{ or } 78\)
12. \(n \div 6 = 14\)  
    \(n = 80, 82, \text{ or } 84\)

Solve each equation. Then, check the solution.

13. \(45 \div n = 5\)  
14. \(100 \div n = 10\)  
15. \(n \div 6 = 7\)  
16. \(n \div 12 = 9\)

17. \(65 \div n = 13\)  
18. \(120 \div n = 30\)  
19. \(n \div 3 = 31\)  
20. \(n \div 4 = 21\)
Problem Solving Skill

Interpret the Remainder

When there is a remainder in a division problem, you need to look at the question to see what is being asked. You may drop the remainder, or round the quotient to the next greater whole number, or you may use the remainder as a fractional part of your answer.

Andy made punch with 48 ounces of apple juice, 36 ounces of grape juice, and 60 ounces of lemon soda. How many 5-ounce servings did he make?

\[ 48 + 36 + 60 = 144 \text{ oz} \]

\[
\begin{array}{c}
28 \text{ r} 4 \\
5)144 \\
-10 \\
44 \\
-40 \\
4
\end{array}
\]

There are 4 ounces left over.
That is not enough for another 5-ounce serving. Drop the remainder.

So, Andy made 28 five-ounce servings.

Solve. Explain how you interpreted the remainder.

1. Mia bought 10 feet of wire for a science project. She divided the wire equally into 3 pieces. How long was each piece of wire?

2. A total of 325 people will be attending a sports banquet. There will be 8 people seated at each table. How many tables will be needed?

3. A total of 175 players signed up for a baseball league. There are 9 teams in the league. If the players are divided among the teams, what is the greatest number of players on any team?

4. Jennie baked 132 cookies. She wants to divide them evenly among her 7 friends. How many cookies will she give to each friend?
Algebra: Patterns in Division

Rick has a 1,313-page book. If he reads 14 pages a day, about how long will it take him to finish reading the book? You divide to find the answer.

\[ \frac{1,313}{14} \]

You can estimate to find the number of days it will take Rick to read the book.

Estimate: 1,313 rounds down to 1,000.
14 rounds down to 10.

\[ \frac{1,000}{10} \]

There are zeros in the dividend and in the divisor. Cancel out one zero in each.

\[ \frac{1,000}{10} = 100 \]

So, it will take Rick about 100 days to read the book.

You can check this estimate by multiplying. Multiply the divisor by the quotient.

\[ 10 \times 100 = 1,000 \]

Find each quotient. Cancel out the zeros if appropriate. Write a multiplication sentence to check. The first one is done for you.

1. \[ \frac{1,500}{30} = 50 \]
   \[ 30 \times 50 = 1,500 \]
2. \[ \frac{560}{70} = \_\_\_ \]
3. \[ \frac{720}{80} = \_\_\_ \]
4. \[ \frac{2,100}{70} = \_\_\_ \]
5. \[ \frac{480}{60} = \_\_\_ \]
6. \[ \frac{2,500}{50} = \_\_\_ \]
7. \[ \frac{36,000}{90} = \_\_\_ \]
8. \[ \frac{24,000}{40} = \_\_\_ \]
9. \[ \frac{5,600}{80} = \_\_\_ \]
Estimate Quotients

Compatible numbers are numbers that are close to the actual numbers and can be divided evenly. They can help you estimate a quotient.

Estimate. \( \frac{42}{1,574} \)

Step 1
Round the divisor. The number 42 rounds to 40. It can also be rounded to 50.

Step 2
Round the dividend. The number 1,574 can be rounded up to 1,600 or rounded down to 1,500.

Step 3
Rewrite the division problem with the compatible numbers, and solve.

\[
\begin{array}{ccc}
\text{40} & & \text{30} \\
40 \div 1,600 & & 50 \div 1,500 \\
\end{array}
\]

So, one estimate of the quotient is 40. A second estimate is 30.

Write two pairs of compatible numbers for each. Give two possible estimates.

1. \( \frac{48}{3,367} \)  
   ____________  
   ____________

2. \( \frac{76}{4,117} \)  
   ____________  
   ____________

3. \( \frac{37}{847} \)  
   ____________  
   ____________

4. \( \frac{54}{2,438} \)  
   ____________  
   ____________

5. \( \frac{68}{4,831} \)  
   ____________  
   ____________

6. \( \frac{73}{26,970} \)  
   ____________  
   ____________
Divide by 2-Digit Divisors

A total of 6,501 people attended the local theater. A movie was shown 20 times during a 5-day period. The same number of people attended each showing except for the first. How many people attended each showing?

**Step 1**

Decide where to place the first digit in the quotient. Are there enough thousands?

No, 6 < 20. Are there enough hundreds? Yes, 65 > 20. The first digit goes in the hundreds place.

**Step 2**

Divide the hundreds. 20 \( \overline{\div} \) 65

Write the 3 in the hundreds place.

Multiply. 20 \( \times \) 3

Subtract. 65 \( - \) 60

Compare. 5 < 20

So, 325 people attended each showing of the movie, with 1 more person, or 326 people, attending the first showing.

Follow the steps above to find each quotient.

1. \( 52 \overline{\div} 6,219 \)
2. \( 81 \overline{\div} 9,017 \)
3. \( 24 \overline{\div} 6,008 \)

4. \( 17 \overline{\div} 92,418 \)
5. \( 32 \overline{\div} 6,850 \)
6. \( 41 \overline{\div} 87,409 \)
Correcting Quotients

Maria collects postcards. She has 389 postcards in her collection. The cards are organized in albums that hold 48 postcards each. How many albums has Maria used?

Divide. 389 ÷ 48

Step 1
Write two pairs of compatible numbers, and estimate the answer.

\[
\begin{array}{c}
40)360 \\
50)400 \\
\end{array}
\]

\[
\begin{array}{c}
9 \\
8 \\
\end{array}
\]

Step 2
Use one of your estimates. The divisor, 48, is closer to 50. Use 8 as the first digit in the quotient.

Step 3
Divide.

\[
\begin{array}{c}
8 \text{ r} 5 \\
\end{array}
\]

Since 5 < 48, the estimate is just right.

So, Maria has 8 full albums and 1 album with only 5 postcards in it.

Use the steps above to find each quotient.

1. 19)67
2. 31)97
3. 48)235
4. 74)975

5. 62)557
6. 27)292
7. 52)509
8. 85)768

9. 75)5,387
10. 49)8,372
11. 65)41,760
12. 54)59,534
Practice Division

Ron’s Record Shop received a shipment of 756 tapes. The tapes were packaged in 28 cartons. Each carton held the same number of tapes. How many tapes were in each carton?

**Step 1**

756 \div 28

Decide where to place the first digit.
Are there enough hundreds? No, 7 < 28.
Place the first digit in the tens place.

**Step 2**

2

Divide the 75 tens.
Multiply. 28 \times 2
Subtract. 75 - 56
Compare. 19 < 28

**Step 3**

27

Divide the 196 ones.
Multiply. 28 \times 7
Subtract. 196 - 196
So, each carton held 27 tapes.

You can use multiplication to check the answer. Multiply the divisor by the quotient. Add any remainder.

28 \times 27 = 756 \quad \text{The answer checks.}

Follow the steps above to find each quotient. Check by multiplying.

1. 17 \div 255
2. 26 \div 396
3. 33 \div 458
4. 49 \div 721

5. 45 \div 6,004
6. 39 \div 72,118
7. 15 \div 497
8. 54 \div 36,565
Problem Solving Strategy: Predict and Test

Rhea has 253 stickers. She has them stored in equal groups in containers and has started a new container with 3 stickers in it. How many containers of stickers does she have? How many stickers are in each container?

Step 1

Subtract the 3 stickers in the new container from the 253 total number of stickers. $253 - 3 = 250$
There are 250 can be divided by 5.

Step 2

Use predict and test to find the number of equal groups in 250. The number ends with 0, so 250 can be divided by 5.

Step 3

Divide. $250 \div 5 = 50$

Check. 

\[
\begin{array}{c}
50 \\
\times 5 \\
\end{array}
\]

\[
\begin{array}{c}
250 \\
\end{array}
\]

\[
\begin{array}{c}
\pm 3 \\
\end{array}
\]

\[
\begin{array}{c}
253 \\
\end{array}
\]

✔ The answer checks.

So, Rhea has 5 containers of stickers with 50 stickers in each container. There are 3 stickers in the new container.

Predict and test to solve.

1. James has 467 bookmarks in his collection. He has them stored in equal groups in boxes. He then starts a new box with 5 bookmarks in it. How many boxes of bookmarks does he have? How many bookmarks are in each box?

2. Nora baked 156 brownies. She is putting them into packages with an equal number of brownies in each. She eats 2 brownies. How many packages does she make? How many brownies are in each package?
Algebra: Patterns in Decimal Division

Kara is dividing $3 equally into 5 boxes. How much money should go into each box?

$3 ÷ 5 = ?

Using a pattern can help you find the exact answer.

Write similar number sentences with zeros added to the dividends. The decimal point shifts one place to the left each time.

3,000 ÷ 5 = 600.0
300 ÷ 5 = 60.0
30 ÷ 5 = 6.0
3 ÷ 5 = 0.6

So, each box gets 0.6, or $0.60.

Complete each number sentence. Look for a pattern.

1. 3,000 ÷ 6 = 500  2. 4,500 ÷ 5 = 900  3. 6,400 ÷ 8 = 800  4. 2,800 ÷ 7 = 400
   ____ ÷ 6 = 50  ____ ÷ 5 = 90  ____ ÷ 8 = 80  280 ÷ 7 = ____
   30 ÷ 6 = ____  45 ÷ 5 = ____  64 ÷ 8 = ____  28 ÷ 7 = ____
   3 ÷ 6 = ____  4.5 ÷ 5 = ____  6.4 ÷ 8 = ____  2.8 ÷ 7 = ____

Use a pattern to write the quotients.

5. 400 ÷ 8 = ____  6. 600 ÷ 4 = ____  7. 800 ÷ 5 = ____
   40 ÷ 8 = ____  60 ÷ 4 = ____  80 ÷ 5 = ____
   4 ÷ 8 = ____  6 ÷ 4 = ____  8 ÷ 5 = ____

8. 1,400 ÷ 7 = ____  9. 13,000 ÷ 5 = ____  10. 2,700 ÷ 9 = ____
   140 ÷ 7 = ____  1,300 ÷ 5 = ____  270 ÷ 9 = ____
   14 ÷ 7 = ____  130 ÷ 5 = ____  27 ÷ 9 = ____
   1.4 ÷ 7 = ____  13 ÷ 5 = ____  2.7 ÷ 9 = ____
**Decimal Division**

You can use a centimeter ruler to help you divide 1.4 by 2.

1 stands for 1 centimeter.

Each space stands for \( \frac{1}{10} \), or 0.1, cm.

**Step 1**

Find 1.4 centimeters on the ruler. Count the number of spaces.

There are 14 spaces.

**Step 2**

Divide the number of spaces by 2.

14 ÷ 2 = 7

Count over 7 spaces.

The seventh space is 0.7 cm.

So 1.4 ÷ 2 = 0.7.

---

Use the centimeter ruler to find the quotient.

1. 2.4 ÷ 6 = _____

2. 2.5 ÷ 5 = _____

3. 1.8 ÷ 3 = _____

4. 2.4 ÷ 4 = _____

5. 2.1 ÷ 7 = _____

6. 1.6 ÷ 8 = _____

7. 3.9 ÷ 3 = _____

8. 3.3 ÷ 3 = _____

9. 0.8 ÷ 8 = _____
Divide Decimals by Whole Numbers

You can use a centimeter ruler to help you divide 3.6 by 2.

1 stands for 1 centimeter. Each space stands for $\frac{1}{10}$ or 0.1 cm.

Step 1
Find 3.6 centimeters on the ruler.

Step 2
There are 3 whole centimeters. Divide them into 2 equal groups. There is 1 centimeter in each group with 1.6 centimeters left over.

Step 3
Count the spaces for the remaining 1.6 cm. There are 16 spaces. Divide them into 2 groups. There are 2 groups of 8 spaces. Each group is 0.8 centimeter.

Step 4
There are 2 equal groups of 1.8 centimeters.

So, $3.6 \div 2 = 1.8$

Use the ruler to find the quotient.

1. $3 \div 3.6$

2. $2 \div 2.8$

3. $2 \div 3.2$

4. $4 \div 4.0$

5. $2 \div 1.2$

6. $4 \div 3.6$
Problem Solving Strategy

**Compare Strategies**

**Problem** Since the school year began, Jill has grown 0.75 inches. Now she measures 58.5 inches. What did she measure when the year began?

Work backward: What information can you use to find out how tall Jill was when the year began? You can start by using the information at the end. Now she is 58.5 inches. Then use the fact that she grew 0.75 inch. Work backward to find out how tall she was at the beginning of the year.

58.5 = current height

0.75 = height she grew

57.75 inches = height at beginning of school year.

You can also use predict and test.

Predict: She was 57 inches. Test: 57 + 0.75 = 57.75; too low

Predict again, using a higher number.

Predict: She was 57.75 inches.

Test: 57.75 + 0.75 = 58.5 inches

**What strategy can you use?**

Work backward

Predict and Test

Solve and write the problem solving strategy you used: work backward or predict and test.

1. Anthony started with his favorite number. Then he subtracted 7 from it. He multiplied this difference by 3 and then added 5. Finally he divided this number by 11. His end result was 1. What was Anthony’s favorite number?

3. The sum of 2 numbers is 40 and their difference is 2. What are the two numbers?

2. Forty-seven baseball players need a ride to the play-off game. Each car has seat belts for 4 players and can make 2 trips. How many cars will be needed?

4. The school spent $438.75 to buy art supplies and gym supplies. The total cost of the art supplies was $230.60. How much was spent on the gym supplies?
Divide to Change a Fraction to a Decimal

Fractions can be written as decimals by dividing the numerator by the denominator.

\[
\frac{\text{numerator}}{\text{denominator}} \rightarrow \text{denominator} \div \text{numerator}
\]

To write \(\frac{3}{5}\) as a decimal, divide 3 by 5.

\[
\frac{3}{5} = 5)3.0 \leftarrow \text{numerator}
\]
\[
\text{denominator} \quad \frac{3.0}{0}
\]

Write as a decimal.

1. \(\frac{3}{50}\)
2. \(\frac{4}{10}\)
3. \(\frac{16}{100}\)
4. \(\frac{3}{4}\)
5. \(\frac{20}{40}\)

6. \(\frac{8}{10}\)
7. \(\frac{15}{20}\)
8. \(\frac{4}{5}\)
9. \(\frac{42}{50}\)
10. \(\frac{10}{25}\)

11. \(\frac{63}{100}\)
12. \(\frac{1}{8}\)
13. \(\frac{1}{4}\)
14. \(\frac{4}{8}\)
15. \(\frac{6}{25}\)

16. \(\frac{3}{8}\)
17. \(\frac{3}{15}\)
18. \(\frac{721}{1,000}\)
19. \(\frac{4}{100}\)
20. \(\frac{5}{8}\)

21. \(\frac{14}{25}\)
22. \(\frac{47}{50}\)
23. \(\frac{8}{1,000}\)
24. \(\frac{7}{8}\)
25. \(\frac{30}{1,000}\)
Algebra: Patterns in Decimal Division

Helen has $2.00, and she is putting $0.50 into each box. How many boxes can she fill?

$2.00 ÷ 0.50 = ?$

Using a pattern can help you find the exact answer.

Notice that moving the decimal point one place to the left for the divisor and the dividend will give the same answer each time.

So, She can fill 4 boxes with $0.50 in each box.

Complete each number sentence. Look for patterns.

1. 240 ÷ 6 = 40
2. 320 ÷ 8 = 40
3. 490 ÷ 7 = 70
4. 540 ÷ 9 = 60
   24.0 ÷ ____ = 40
   32.0 ÷ ____ = 40
   49.0 ÷ ____ = 70
   54.0 ÷ ____ = 60
   2.4 ÷ 0.06 = ____
   3.2 ÷ 0.08 = ____
   4.9 ÷ 0.07 = ____
   5.4 ÷ 0.09 = ____

Use patterns to write the quotients.

5. 36 ÷ 6 = _____
6. 42 ÷ 7 = _____
7. 54 ÷ 6 = _____
   3.6 ÷ 0.6 = _____
   4.2 ÷ 0.7 = _____
   5.4 ÷ 0.6 = _____
   0.36 ÷ 0.06 = _____
   0.42 ÷ 0.07 = _____
   0.54 ÷ 0.06 = _____

8. 32 ÷ 4 = _____
9. 56 ÷ 8 = _____
10. 63 ÷ 7 = _____
    3.2 ÷ 0.4 = _____
    5.6 ÷ 0.8 = _____
    6.3 ÷ 0.7 = _____
    0.32 ÷ 0.04 = _____
    0.56 ÷ 0.08 = _____
    0.63 ÷ 0.07 = _____

11. 28 ÷ 4 = _____
12. 24 ÷ 6 = _____
13. 48 ÷ 6 = _____
    2.8 ÷ 0.4 = _____
    2.4 ÷ 0.6 = _____
    4.8 ÷ 0.6 = _____
    0.28 ÷ 0.04 = _____
    0.24 ÷ 0.06 = _____
    0.48 ÷ 0.06 = _____
Divide with Decimals

Use the following rules to divide a decimal by another decimal:

1.) Move the \underline{decimal point} in the divisor as far **right** as possible.

2.) Move the \underline{decimal point} in the dividend to the **right** the same number of places as you did in the divisor.

3.) Put the \underline{decimal point} in the quotient directly above the new decimal point in the dividend.

4.) Divide the numbers to obtain the quotient.

Example:

Move the \underline{decimal point} 2 places to the **right**.

Divide the numbers to obtain the quotient.

\[ 0.07 \div 0.42 = 7.042. \]

Find the quotient. Check by multiplying.

1. \(3.2 \div 0.4 = \underline{\hspace{2cm}}\) \hspace{2cm} 2. \(6.4 \div 0.8 = \underline{\hspace{2cm}}\)

3. \(0.21 \div 0.07 = \underline{\hspace{2cm}}\) \hspace{2cm} 4. \(0.50 \div 0.25 = \underline{\hspace{2cm}}\)

5. \(3.9 \div 1.3 = \underline{\hspace{2cm}}\) \hspace{2cm} 6. \(0.96 \div 0.24 = \underline{\hspace{2cm}}\)

7. \(2.4 \div 0.4 = \underline{\hspace{2cm}}\) \hspace{2cm} 8. \(0.49 \div 0.07 = \underline{\hspace{2cm}}\)
Decimal Division

When dividing a decimal by another decimal, you must change the divisor to a whole number by multiplying the divisor by 10 or 100. Whatever number you use to multiply the divisor, you must also use to multiply the dividend. Your divisor is a whole number and your dividend is larger by the same amount.

Divide 52.36 by 0.28.

Write the division problem on graph paper.

Move the decimal point in the divisor by multiplying by 100. 0.28 becomes 28.0.

Move the decimal point in the dividend by multiplying it by 100. 52.36 becomes 5236.0.

Divide as if you were working with whole numbers.

Find the quotient. Check by multiplying.

1. \(0.5 \div 9.25\)  
   Check:

2. \(5.2 \div 4.524\)  
   Check:

3. \(0.8 \div 0.896\)  
   Check:

4. \(0.56 \div 24.472\)  
   Check:

5. \(24.78 \div 0.3 = \)  
   Check:

6. \($39.00 \div 0.52 = \)  
   Check:

7. \(9.144 \div 0.36 = \)  
   Check:

8. \($1.84 \div 0.04 = \)  
   Check:
Problem Solving Skill

Choose the Operation

Ricardo and his two friends raise small animals. Ricardo buys rabbits, hamsters, mice, and gerbils. If Ricardo and his friends each take an equal number of animals, how many animals will each person get?

There are 15 rabbits for 3 people.

Should you multiply? or Should you divide?

\[ 15 \times 3 = 45 \]

\[ 15 \div 3 = 5 \]

Which answer makes more sense? Since they bought only 15 rabbits, 5 rabbits each makes the most sense. You should divide.

For Problems 1–6, use the table to solve each problem. Name the operation you used.

<table>
<thead>
<tr>
<th>Type of Animal</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rabbits</td>
<td>15</td>
</tr>
<tr>
<td>Hamsters</td>
<td>27</td>
</tr>
<tr>
<td>Mice</td>
<td>36</td>
</tr>
<tr>
<td>Gerbils</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type of Food</th>
<th>Amount (in pounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rabbit Food</td>
<td>186.3</td>
</tr>
<tr>
<td>Hamster Food</td>
<td>53.1</td>
</tr>
<tr>
<td>Mouse Food</td>
<td>26.9</td>
</tr>
<tr>
<td>Gerbil Food</td>
<td>12.6</td>
</tr>
</tbody>
</table>

1. Ricardo and his two friends purchase animal food. They share what they buy equally. What is Ricardo’s share of the rabbit food?

2. Ricardo buys the same amount of gerbil food each month for 5 months. How much gerbil food does Ricardo buy?

3. Ricardo pays $1.25 per pound for a month’s worth of gerbil food. How much does the gerbil food cost in all?

4. Ricardo spent $37.17 buying hamster food. What was the cost per pound for the hamster food?

5. How much animal food do Ricardo and his friends buy in all?

6. What is Ricardo’s share of the hamster food?
Divisibility

The rules for divisibility by 3 and 9 are special. They depend on finding the sum of the digits.

- A number is divisible by 3 if the sum of the digits of the number is divisible by 3.
- A number is divisible by 9 if the sum of the digits of the number is divisible by 9.

1. Decide if 615 is divisible by 3.
   a. What is the sum of the digits 6, 1, and 5? ____________
   b. Is 12 divisible by 3? ____________
   c. Is 615 divisible by 3? ____________

2. Decide if 615 is divisible by 9.
   a. What is the sum of the digits 6, 1, and 5? ____________
   b. Is 12 divisible by 9? ____________
   c. Is 615 divisible by 9? ____________

Tell if each number is divisible by 3 or 9.

3. 90
   _________

4. 315
   _________

5. 390
   _________

6. 405
   _________

7. 75
   _________

8. 4,770
   _________

9. 320
   _________

10. 3,705
    _________

11. 801
    _________

12. 408
    _________

13. 117
    _________

14. 490
    _________

15. 81
    _________

16. 906
    _________

17. 432
    _________

18. 235
    _________

19. 123
    _________

20. 684
    _________

21. 963
    _________

22. 91
    _________
Multiples and Least Common Multiples

Sam and Mary love to count. Sam counts by 3’s and Mary counts by 4’s.

Sam  Mary
3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, . . . 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, . . .

Sam and Mary both say the numbers 12 and 24. These numbers are called the **common multiples** of 3 and 4. The first common multiple is 12, so it is called the **least common multiple** of 3 and 4.

---

List the first 6 multiples of the number.

1. 2
2. 5
3. 6
4. 7
5. 8
6. 9
7. 10
8. 11
9. 12

Find the first 2 common multiples of each pair of numbers.

10. 2 and 5
11. 4 and 8
12. 6 and 8
13. 4 and 12
14. 3 and 8
15. 6 and 9
16. 5 and 8
17. 3 and 7
Greatest Common Factor

You can find the greatest common factor of two numbers. It is the greatest factor that the two numbers have in common.

Find the greatest common factor of 9 and 15.

Step 1
List all the factors of each number.
9: 1, 3, 9
15: 1, 3, 5, 15

Step 2
Note the common factors.
The common factors of 9 and 15 are 1 and 3.

Step 3
Which factor is greater?
3 is greater than 1.

So, the greatest common factor of 9 and 15 is 3.

Use the factors given to find the greatest common factor (GCF) for each pair of numbers.

1. 10: 1, 2, 5, 10
   25: 1, 5, 25
   GCF

2. 18: 1, 2, 3, 6, 9, 18
   21: 1, 3, 7, 21
   GCF

3. 28: 1, 2, 4, 7, 14, 28
   35: 1, 5, 7, 35
   GCF

4. 21: 1, 3, 7, 21
   49: 1, 7, 49
   GCF

List the factors of each number. Write the greatest common factor (GCF) for each pair of numbers. The first one is done for you.

5. 8
   12
   GCF

6. 6
   24
   GCF

7. 9
   27
   GCF

8. 4
   14
   GCF
Problem Solving Skill

Identify Relationships

Identifying relationships can help you solve some word problems.

There is a relationship between the product of two numbers and the product of their least common multiple (LCM) and greatest common factor (GCF).

Example:
• Find the relationship between the product of 6 and 9, and the product of their LCM and GCF.

  The LCM of 6 and 9 is 18.
  The GCF of 6 and 9 is 3.
  
  \[ 6 \times 9 = 54 \quad \text{LCM} \times \text{GCF} = 18 \times 3 = 54 \]

So, the product of two numbers is equal to the product of their LCM and GCF.

Use the relationship between the given numbers to complete the table.

<table>
<thead>
<tr>
<th>First Number</th>
<th>Second Number</th>
<th>Product of Numbers</th>
<th>Product of LCM and GCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>15</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td></td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td>36</td>
</tr>
</tbody>
</table>

Use the relationships between the given numbers to solve.

1. The product of the LCM and GCF of 4 and another number is 36. What is the other number?

2. The product of two numbers is 98. The GCF of the two numbers is 7. What is their LCM?

3. The product of the LCM and GCF of two numbers is 55, and neither of the numbers is 1. What are the two numbers?

4. The product of two numbers is 320. The GCF of the two numbers is 4, and one of the numbers is 16. What is the other number?
Prime and Composite Numbers

You can use squares to see if a number is prime or composite.

A **prime number** has exactly two factors, 1 and the number itself.  

A **composite number** has more than two factors.

Is the number 5 prime or composite?  
Is the number 8 prime or composite?

Only 2 arrangements of squares are possible (5 × 1, 1 × 5). The number 5 has exactly two factors, so it is a prime number.

More than 2 arrangements of squares are possible (8 × 1, 1 × 8, 4 × 2, 2 × 4). The number 8 has more than two factors, so it is a composite number.

Draw squares to see if each number is prime or composite.  
Write *prime* or *composite*.

1. 7

2. 6

Write the possible arrangements of squares for each number. Then write *prime* or *composite*. The first one is done for you.

3. 4 1 × 4, 4 × 1, 2 × 2; composite

4. 9

5. 10

6. 11

7. 12

8. 13
Introduction to Exponents

Exponents are also called “powers.”

\[ 10 \times 10 = 10^2 \]
\[ 10^2 = 10 \text{ to the power of } 2 \]

\[ 10 \times 10 \times 10 = 10^3 \]
\[ 10^3 = 10 \text{ to the power of } 3 \]

Show 10 to the power of 8 in four different ways.

<table>
<thead>
<tr>
<th>Exponent Form</th>
<th>Expanded Form</th>
<th>Standard Form</th>
<th>Word Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^8 )</td>
<td>( 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 )</td>
<td>( 10^8 = 100,000,000 )</td>
<td>One hundred million</td>
</tr>
</tbody>
</table>

Write in expanded form.

1. 100
2. 10,000
3. 1,000
4. 100,000

5. \( 10^6 \)
6. \( 10^3 \)
7. \( 10^5 \)
8. \( 10^7 \)

Write in exponent form.

9. 100,000
10. 10,000
11. 10,000,000
12. 1,000

13. 10 to the power of 3
14. 10 to the power of 7
15. 10 to the power of 9
16. 10 to the power of 10
Evaluate Expressions with Exponents

What number does $6^5$ represent?

6 is called the base. The 5 is called the exponent. The exponent tells you how many times the base is used as a factor.

$6^5 = 6 \times 6 \times 6 \times 6 \times 6 = 7,776$

$4^3 = 4 \times 4 \times 4 = 64$

4 is the base and 3 is the exponent.

Write the base and the exponent.

1. $4^6$
   Base: __________  Exponent: __________
2. $6^4$
   Base: __________  Exponent: __________
3. $9^{18}$
   Base: __________  Exponent: __________
4. $5^7$
   Base: __________  Exponent: __________

Write the equal factors.

5. $9^9$
   __________
6. $6^7$
   __________
7. $3^9$
   __________
8. $12^6$
   __________

9. $14^2$
   __________
10. $8^{10}$
    __________
11. $11^{11}$
    __________
12. $24^4$
    __________

Find the value.

13. $4^6$
   __________
14. $6^4$
   __________
15. $9^3$
   __________
16. $5^2$
   __________
Exponents and Prime Factors

You can think about prime factorization as a series of division problems.

Begin with the number you need to factor: 48

What is the least possible prime number that divides 48? 2

Keep dividing by prime divisors until you get 1 as a quotient.

1. Divide 2 into 48.
   \[ \frac{24}{2} = 12 \]
2. Is the quotient 1? No.
   \[ \frac{12}{2} = 6 \]

Repeat the process.
3. Is the quotient 1? No.
   \[ \frac{6}{2} = 3 \]

Repeat the process.
4. Is the quotient 1? No.
   \[ \frac{3}{2} = 1 \]

Stop.

Write the prime divisors as factors of 48.

\[ 48 = 2 \times 2 \times 2 \times 2 \times 3 \]

Use what you know about exponents to write the factors.

\[ 48 = 2^4 \times 3 \]

Write the prime factorization of the number. Use exponents when possible.

1. 12
   \[ \underline{2} \times 2 \times 3 \]
2. 24
   \[ \underline{2} \times 2 \times 2 \times 2 \times 3 \]
3. 28
   \[ \underline{2} \times 2 \times 7 \]
4. 45
   \[ \underline{3} \times 3 \times 5 \]
5. 36
   \[ \underline{2} \times 2 \times 3 \times 3 \]
6. 125
   \[ 5 \times 5 \times 5 \]
7. 256
   \[ 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \]
8. 81
   \[ 3 \times 3 \times 3 \times 3 \]
# Relate Decimals to Fractions

You can write a fraction or a decimal to tell what part is shaded.

<table>
<thead>
<tr>
<th>Model</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Read</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Fraction Model" /></td>
<td>4 shaded parts [\frac{\text{4}}{100 \text{ parts}}]</td>
<td>0.04</td>
<td>four hundredths</td>
</tr>
<tr>
<td><img src="image2" alt="Fraction Model" /></td>
<td>25 shaded parts [\frac{\text{25}}{100 \text{ parts}}]</td>
<td>0.25</td>
<td>twenty-five hundredths</td>
</tr>
</tbody>
</table>

Complete the table.

<table>
<thead>
<tr>
<th>Model</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Read</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Fraction Model" /></td>
<td>1 shaded parts [\frac{\text{1}}{\text{parts}}]</td>
<td>OT H</td>
<td></td>
</tr>
<tr>
<td><img src="image4" alt="Fraction Model" /></td>
<td></td>
<td>OT H</td>
<td></td>
</tr>
<tr>
<td><img src="image5" alt="Fraction Model" /></td>
<td></td>
<td>OT H</td>
<td></td>
</tr>
<tr>
<td><img src="image6" alt="Fraction Model" /></td>
<td></td>
<td>OT H</td>
<td></td>
</tr>
<tr>
<td><img src="image7" alt="Fraction Model" /></td>
<td></td>
<td>OT H</td>
<td></td>
</tr>
</tbody>
</table>
**Equivalent Fractions**

You can use different fractions to name the same amount. Fractions that name the same amount are called **equivalent fractions**.

You can find equivalent fractions in three ways.

**Use a number line.**

Use the number lines to find out if the fractions are equivalent. Write yes or no.

1. \( \frac{1}{4} = \frac{3}{12} \) ________

2. \( \frac{8}{12} = \frac{3}{4} \) ________

Multiply both the numerator and the denominator by the same number.

\[
\frac{1}{3} = \frac{1 \times 3}{3 \times 3} = \frac{3}{9}
\]

The fraction \( \frac{1}{3} \) names the same amount as \( \frac{3}{9} \), so they are equivalent fractions.

Divide both the numerator and the denominator by the same number.

\[
\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}
\]

The fractions \( \frac{6}{8} \) and \( \frac{3}{4} \) are equal, so they are equivalent fractions.

---

Use the number lines to find out if the fractions are equivalent.
Write yes or no.

1. \( \frac{1}{4} = \frac{3}{12} \) ________

2. \( \frac{8}{12} = \frac{3}{4} \) ________

Multiply both the numerator and the denominator to name an equivalent fraction.

3. \( \frac{3}{8} = \frac{3 \times 2}{8 \times 2} \)

4. \( \frac{2}{3} = \frac{2 \times 5}{3 \times 5} \)

5. \( \frac{1}{7} = \frac{1 \times 4}{7 \times 4} \)

6. \( \frac{4}{5} = \frac{4 \times 3}{5 \times 3} \)

Divide both the numerator and the denominator to name an equivalent fraction.

7. \( \frac{12}{16} = \frac{12 \div 4}{16 \div 4} \)

8. \( \frac{7}{28} = \frac{7 \div 7}{28 \div 7} \)

9. \( \frac{10}{15} = \frac{10 \div 5}{15 \div 5} \)

10. \( \frac{16}{24} = \frac{16 \div 8}{24 \div 8} \)
Compare and Order Fractions

The three fractions $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{2}{6}$ are arguing about who is the largest.

You can settle the argument by finding a common multiple for the denominators.

**Step 1**

Find the product of all three denominators.

$3 \times 4 \times 6 = 72$

$72$ is a common multiple.

Use it for the denominator.

**Step 2**

Rename each fraction so that $72$ is the denominator.

$\frac{2}{3} \times \frac{24}{24} = \frac{48}{72}$

$\frac{3}{4} \times \frac{18}{18} = \frac{54}{72}$

$\frac{2}{6} \times \frac{12}{12} = \frac{24}{72}$

**Step 3**

Compare the numerators. Put them in order from least to greatest.

$\frac{24}{72} < \frac{48}{72} < \frac{54}{72}$

$\frac{2}{6} < \frac{2}{3} < \frac{3}{4}$

So, $\frac{3}{4}$ is the largest fraction.

---

**Find the product of the denominators.**

1. $\frac{2}{5}, \frac{3}{4}, \frac{5}{7}$

2. $\frac{2}{9}, \frac{1}{3}, \frac{1}{2}$

3. $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}$

---

**Rename the fractions by using a common denominator.**

4. $\frac{2}{5}, \frac{3}{4}, \frac{5}{7}$

5. $\frac{2}{9}, \frac{1}{3}, \frac{1}{2}$

6. $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}$

---

**Compare and order from least to greatest.**

7. $\frac{2}{5}, \frac{3}{4}, \frac{5}{7}$

8. $\frac{2}{9}, \frac{1}{3}, \frac{1}{2}$

9. $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}$

---
Simplest Form

You can use fraction bars to find the simplest form of a fraction.

Find the simplest form for \(\frac{3}{12}\).

**Step 1** Model \(\frac{3}{12}\) with fraction bars.

**Step 2** Line up other fraction bars to find all the equivalent fractions for \(\frac{3}{12}\). You can see that \(\frac{2}{8}\) and \(\frac{1}{4}\) are equivalent fractions for \(\frac{3}{12}\).

**Step 3** The equivalent fraction that has the largest fraction bar possible is in the simplest form.

So, \(\frac{1}{4}\) is the simplest form of \(\frac{3}{12}\).

Use the fraction bar outlines below to model each fraction and equivalent fractions. Divide the outline into equal parts or keep it whole. Write the fraction in its simplest form.

1. \(\frac{9}{12}\)  

   [Fraction bar outline]  

   ____ Simplest form ____

2. \(\frac{4}{12}\)  

   [Fraction bar outline]  

   ____ Simplest form ____
Understand Mixed Numbers

John drank \(2\frac{3}{4}\) cartons of milk with his lunch.

The number \(2\frac{3}{4}\) is a mixed number. A **mixed number** is made up of a whole number and a fraction.

In the mixed number \(2\frac{3}{4}\), the whole number 2 represents two whole cartons of milk.

In the mixed number \(2\frac{3}{4}\), the fraction \(\frac{3}{4}\) represents a part of another carton.

You can divide all three cartons into 4 equal parts to show how many fourths John drank.

There are 11 shaded parts. Each part is \(\frac{1}{4}\) carton.

So, John drank \(\frac{11}{4}\), or \(2\frac{3}{4}\), cartons of milk.

---

Write both a fraction and a mixed number for each figure.

1. 

2. 

3. 

4. 

5. 

6.
Problem Solving Strategy

Make a Model

Trisha spent $\frac{3}{4}$ hour on math homework, $\frac{3}{8}$ hour on science, and $\frac{1}{2}$ hour on language arts. Which homework did she spend the most time on?

You can make a model to solve this problem.

**Step 1** For each fraction, draw a box. Shade the box to show the fraction.

**Step 2** Find the LCD, and divide each box into that many equal parts.

The LCD is 8.

**Step 3** Compare the numerators. $\frac{6}{8}$ is the greatest fraction.

So, Trisha spent the most time on math homework.

Make a model to solve.

1. Joe loves to cook. Last weekend he used flour in three different recipes. The amounts were $\frac{3}{4}$ cup, $\frac{2}{4}$ cup, and $\frac{1}{4}$ cup. What was the least amount called for?

2. Karen walked $\frac{5}{6}$ of a mile from her house to a friend's house. Joe walked $\frac{7}{12}$ of a mile to his friend's house. Who walked a greater distance?

3. Nick bought $\frac{2}{3}$ pound ground beef, $\frac{11}{12}$ pound ground turkey, and $\frac{3}{4}$ pound ground veal. Which meat did he buy the most of?

4. In the store display, $\frac{2}{5}$ of the T-shirts were yellow and $\frac{1}{4}$ were blue. Were there more yellow or blue T-shirts?
Add and Subtract Like Fractions

The denominators must be the same when adding or subtracting fractions.

Add $\frac{2}{6} + \frac{1}{6}$.

**Step 1**
Are the denominators the same? Yes.

**Step 2**
Add the numerators. The denominator stays the same.

\[
\begin{align*}
\frac{2}{6} & \quad \leftarrow \quad 2 \text{ sixths} \\
\frac{1}{6} & \quad \leftarrow \quad + \quad 1 \text{ sixth} \\
\hline
\frac{3}{6} & = \frac{1}{2}
\end{align*}
\]

So, $\frac{2}{6} + \frac{1}{6} = \frac{1}{2}$.

To subtract like fractions, subtract the numerators. Remember, the denominator stays the same. Then write the difference over the denominator.

Find the sum or difference. Write the answer in simplest form.

1. $\frac{1}{5} + \frac{2}{5}$
2. $\frac{3}{7} + \frac{2}{7}$
3. $\frac{4}{9} + \frac{2}{9}$
4. $\frac{8}{9} - \frac{7}{9}$
5. $\frac{7}{8} - \frac{1}{8}$
6. $\frac{9}{12} - \frac{5}{12}$
7. $\frac{2}{6} + \frac{3}{6}$
8. $\frac{1}{8} + \frac{3}{8}$
9. $\frac{6}{10} + \frac{3}{10}$
10. $\frac{6}{8} - \frac{1}{8}$
11. $\frac{4}{6} - \frac{1}{6}$
12. $\frac{7}{14} - \frac{4}{14}$
**Add Unlike Fractions**

Use fraction bars to add fractions.

Show $\frac{1}{3} + \frac{1}{6}$ with fraction bars.

Now, find like fraction bars that fit exactly under the sum $\frac{1}{3} + \frac{1}{6}$.

So, three sixth bars fit under the sum.

$\frac{3}{6}$ equals $\frac{1}{2}$ in simplest form.

So, $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$.

Use fraction bars to find the sum.

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<td>14</td>
<td>$\frac{1}{6} + \frac{2}{3}$</td>
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</table>
Subtract Unlike Fractions

Use fraction bars to subtract fractions.

Show \( \frac{5}{6} - \frac{1}{3} \) with fraction bars.

Now, find like fraction bars that fit exactly under the difference \( \frac{5}{6} - \frac{1}{3} \).

So, three sixth bars fit under the difference.

\( \frac{3}{6} \) equals \( \frac{1}{2} \) in simplest form.

So, \( \frac{5}{6} - \frac{1}{3} = \frac{1}{2} \).

Use fraction bars to find the difference.

1. \( \frac{1}{12} \) \( \frac{1}{12} \) \( \frac{1}{12} \) \( \frac{1}{12} \) \( \frac{1}{12} \) \( \frac{1}{12} \) \( \frac{1}{12} \) \( \frac{1}{12} \)

\( \frac{1}{3} \) ?

2. \( \frac{1}{10} \) \( \frac{1}{10} \) \( \frac{1}{10} \) ?

3. \( \frac{1}{6} \) \( \frac{1}{6} \) \( \frac{1}{6} \) \( \frac{1}{6} \)

\( \frac{1}{2} \) ?

4. \( \frac{1}{8} \) \( \frac{1}{8} \) \( \frac{1}{8} \) \( \frac{1}{8} \) \( \frac{1}{8} \) \( \frac{1}{8} \) \( \frac{1}{8} \) \( \frac{1}{8} \)

\( \frac{1}{4} \) ?

5. \( \frac{1}{3} \) \( \frac{1}{3} \)

\( \frac{1}{6} \) \( \frac{1}{6} \) \( \frac{1}{6} \) ?

6. \( \frac{1}{10} \) \( \frac{1}{10} \) \( \frac{1}{10} \) ?

7. \( \frac{3}{4} - \frac{1}{2} = \) ________

8. \( \frac{6}{8} - \frac{1}{4} = \) ________

9. \( \frac{2}{3} - \frac{1}{2} = \) ________
**Estimate Sums and Differences**

You can round fractions to 0, to $\frac{1}{2}$, or to 1 to estimate sums and differences.

Estimate the sum $\frac{3}{5} + \frac{8}{9}$.

**Step 1** Find $\frac{3}{5}$ on the number line. Is it closest to 0, $\frac{1}{2}$, or 1? The fraction $\frac{3}{5}$ is closest to $\frac{1}{2}$.

**Step 2** Find $\frac{8}{9}$ on the number line. Is it closest to 0, $\frac{1}{2}$, or 1? The fraction $\frac{8}{9}$ is closest to 1.

**Step 3** To estimate the sum $\frac{3}{5} + \frac{8}{9}$, add the two rounded numbers.

So, $\frac{3}{5} + \frac{8}{9}$ is about $1\frac{1}{2}$.

Use the number lines to estimate whether each fraction is closest to 0, to $\frac{1}{2}$, or to 1. Then find the sum or difference. The first one is done for you.

1. $\frac{4}{6} + \frac{1}{8}$
   
   $\frac{1}{2} + 0$

2. $\frac{2}{6} + \frac{7}{8}$
   
   __ + __

3. $\frac{5}{6} - \frac{3}{8}$
   
   __ - __

4. $\frac{4}{6} + \frac{3}{8}$
   
   __ + __

5. $\frac{7}{8} - \frac{5}{6}$
   
   __ - __

6. $\frac{1}{6} + \frac{7}{8}$
   
   __ + __
Use Least Common Denominators

Tim and Barbara shared a pizza. Tim ate \( \frac{1}{3} \) of the pizza, and Barbara ate \( \frac{4}{9} \).

How much of the pizza did they eat in all?

You add \( \frac{1}{3} + \frac{4}{9} \) to answer this question.

Use the least common denominator to add the unlike fractions.

The least common denominator is the least common multiple of the denominators.

**Step 1:** Find the least common multiple of the denominators.

**Step 2:** Rename each fraction, using the least common denominator.

**Step 3:** Add the like fractions.

**Step 4:** This sum is already in simplest form.

So, Barbara and Tim ate \( \frac{7}{9} \) of the pizza.

To subtract unlike fractions, follow steps 1 and 2. Then subtract and simplify.

Find the least common denominator.

1. \( \frac{1}{3} + \frac{1}{5} \)
   
2. \( \frac{2}{5} + \frac{1}{2} \)
   
3. \( \frac{3}{6} + \frac{3}{4} \)
   
4. \( \frac{7}{9} + \frac{5}{6} \)
   
Find the sum or difference. Write the answer in simplest form.

5. \( \frac{5}{6} + \frac{3}{4} = \) _____

6. \( \frac{7}{12} - \frac{1}{4} = \) _____

7. \( \frac{6}{10} + \frac{3}{5} = \) _____

8. \( \frac{5}{6} - \frac{7}{12} = \) _____

9. \( \frac{1}{2} + \frac{9}{12} = \) _____

10. \( \frac{3}{4} - \frac{1}{3} = \) _____
Add and Subtract Unlike Fractions

When you add or subtract two fractions with unlike denominators, you need to make the denominators the same. Find the least common denominator (LCD), and change the fractions to like fractions with that denominator.

Add. \( \frac{2}{3} + \frac{1}{4} = \text{n} \)

**Step 1**
Find the multiples of both denominators to determine the LCM.

- 3 = 3, 6, 9, 12, . . .
- 4 = 4, 8, 12, 16, . . .

The LCM of 3 and 4 is 12. So, the LCD of \( \frac{2}{3} \) and \( \frac{1}{4} \) is 12.

**Step 2**
Use the LCD to make like fractions. Multiply the numerator and denominator by the same number.

\[
\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}
\]

\[
\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}
\]

Add the fractions.

\[
\frac{8}{12} + \frac{3}{12} = \frac{11}{12}
\]

So, \( n = \frac{11}{12} \).

**Step 3**
Add the fractions.

So, the sum of \( \frac{2}{3} + \frac{1}{4} = \frac{11}{12} \). This answer is in simplest form.

To subtract fractions with unlike denominators, follow these 3 steps. However, in Step 3, subtract the fractions and write the answer in simplest form.

Write like fractions. Then find the sum or difference. Write the answer in simplest form.

1. \( \frac{1}{3} = \frac{1 \times \bcancel{3}}{3 \times \bcancel{3}} = \frac{\bcancel{1}}{\bcancel{3}} = \frac{1}{3} \)

\[
+ \frac{4}{9} = \frac{\bcancel{1}}{\bcancel{3}} + \frac{4}{9}
\]

Simplest form: \( \frac{1}{3} + \frac{4}{9} \)

2. \( \frac{1}{2} = \frac{1 \times \bcancel{2}}{2 \times \bcancel{2}} = \frac{\bcancel{1}}{\bcancel{2}} = \frac{1}{2} \)

\[
- \frac{2}{5} = \frac{1 \times \bcancel{5}}{5 \times \bcancel{5}} = \frac{\bcancel{1}}{\bcancel{5}} = \frac{1}{5}
\]

Simplest form: \( \frac{1}{2} - \frac{2}{5} \)

3. \( \frac{3}{9} = \frac{3 \times \bcancel{3}}{9 \times \bcancel{3}} = \frac{\bcancel{3}}{\bcancel{9}} = \frac{1}{3} \)

\[
+ \frac{1}{6} = \frac{1 \times \bcancel{6}}{6 \times \bcancel{6}} = \frac{\bcancel{1}}{\bcancel{6}} = \frac{1}{6}
\]

Simplest form: \( \frac{1}{3} + \frac{1}{6} \)
Problem Solving Strategy

Work Backward

The students in Jason’s class started measuring their heights at the beginning of January. By March 1, Jason had grown \( \frac{3}{4} \) inch. In February, Jason grew \( \frac{3}{8} \) inch. How much did he grow in January?

You can solve the problem by working backward.

Start with the amount he had grown by March 1, and subtract the amount he grew in February.

Find \( \frac{3}{4} - \frac{3}{8} \) by using the LCD method.

The LCD of 4 and 8 is 8. Change each fraction into eighths, and subtract the numerators.

\[
\frac{3}{4} \times \frac{2}{2} = \frac{6}{8} \quad \frac{3}{8} \times \frac{1}{1} = \frac{3}{8} \quad \frac{6}{8} - \frac{3}{8} = \frac{3}{8}
\]

So, Jason grew \( \frac{3}{8} \) inch in January.

Work backward to solve.

1. Paula is in Jason’s class. By March 1, she had grown \( \frac{7}{8} \) inch. In February, she grew \( \frac{1}{4} \) inch. How much did she grow in January?

2. Sid is in Jason’s class. By April 1, he had grown \( \frac{15}{16} \) inch. In March, he grew \( \frac{1}{8} \) inch, and in February, he grew \( \frac{3}{8} \) inch. How much did he grow in January?

3. Harry started the day by trading 5 of his comic books for 7 of Jenny’s. Next, he bought 8 at the store. Then he gave Tom 9 comic books. Harry came home with 12 comic books. How many did Harry start the day with?

4. Wesley started with his favorite number. Then he subtracted 7 from it. He multiplied this difference by 3 and then added 5. Finally, he divided this number by 11. His end result was 1. What is Wesley’s favorite number?
Add Mixed Numbers

Fred and Gregg are going to put up a tent. They need two pieces of rope to secure the tent. One has to be \(3\frac{1}{4}\) feet long and the other \(2\frac{1}{2}\) feet long. How much rope do they need?

To find the answer, you must add \(3\frac{1}{4} + 2\frac{1}{2}\).

You can add mixed numbers by following these steps.

**Step 1**
Add the whole numbers. \(3 + 2 = 5\)

**Step 2**
Find the LCD. Write equivalent fractions. Add the fractions.

- multiples of 4: 4, 8, 12
  \[\frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}\]
- multiples of 2: 2, 4, 6
  \[\frac{1}{4} = \frac{1 \times 2}{4 \times 1} \quad \frac{1}{2} = \frac{2}{2 \times 2} = \frac{2}{4}\]

**Step 3**
Add the sum of the whole numbers to the sum of the fractions. Write the answer in simplest form if needed.

\[5 + \frac{3}{4} = \frac{23}{4}\]

So, \(3\frac{1}{4} + 2\frac{1}{2} = 5\frac{3}{4}\).

Find the sum in simplest form.

1. \(\frac{3}{8} + 2\frac{1}{8} = \frac{5}{8}\)
2. \(\frac{6}{5} + 2\frac{1}{12} = \frac{11}{12}\)
3. \(\frac{4}{4} + 2\frac{1}{4} = \frac{11}{4}\)
4. \(\frac{5}{7} + 1\frac{3}{7} = \frac{12}{7}\)
5. \(\frac{7}{2} + 2\frac{1}{3} = \frac{11}{3}\)
6. \(\frac{4}{5} + 2\frac{1}{10} = \frac{13}{10}\)
7. \(\frac{4}{2} + 3\frac{3}{8} = \frac{11}{8}\)
8. \(\frac{3}{4} + 2\frac{1}{8} = \frac{11}{8}\)
Subtract Mixed Numbers

Sonia cut out a pattern for a new skirt from the \(3 \frac{1}{2}\) yards of material she bought. The pattern used \(2 \frac{1}{3}\) yards. How much material was left?

You can answer the question by subtracting, \(3 \frac{1}{2} - 2 \frac{1}{3}\).

To subtract mixed numbers, follow these steps.

**Step 1**
Find the LCD of the fractions by listing the multiples of each number.

Multiples of 2: \(2, 4, 6, 8, 10\)
Multiples of 3: \(3, 6, 9, 12, 15\)

Since 6 is the first common multiple, it is the least common multiple.

**Step 3**
Subtract the fractions.

\[
3 \frac{1}{2} - 2 \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}
\]

**Step 2**
Change the fractions into like fractions with 6 as the denominator.

\[
\frac{1}{2} \times \frac{3}{3} = \frac{3}{6} \quad \frac{1}{3} \times \frac{2}{2} = \frac{2}{6}
\]

So, Sonia has \(1 \frac{1}{6}\) yards left.

Subtract. Write the answer in simplest form.

1. \(4 \frac{4}{5} = \frac{4}{8} \quad \frac{10}{10}\)
   
   \[-1 \frac{1}{10} = -1 \frac{1}{10}\]

2. \(6 \frac{2}{3} = \frac{6}{4} \quad \frac{6}{6}\)
   
   \[-4 \frac{1}{6} = -4 \frac{1}{6}\]

3. \(7 \frac{3}{4} = \frac{7}{9} \quad \frac{12}{12}\)
   
   \[-4 \frac{5}{12} = -4 \frac{5}{12}\]

4. \(8 \frac{1}{3} = \frac{8}{12}\)
   
   \[-1 \frac{3}{4} = -1 \frac{3}{12}\]

5. \(2 \frac{7}{8} = \frac{2}{8}\)
   
   \[-1 \frac{1}{2} = -1 \frac{4}{8}\]

6. \(6 \frac{7}{9}\)
   
   \[-4 \frac{2}{3} = -4 \frac{6}{9}\]
Subtraction with Renaming

Wayne had $4\frac{1}{4}$ feet of rope. He gave $2\frac{2}{3}$ feet to his friend. How much rope did he have left?

You can answer the question by subtracting, $4\frac{1}{4} - 2\frac{2}{3}$.

To subtract mixed numbers, follow these steps.

**Step 1**
Find the LCD.

Multiples of 4: 4, 8, 12, 16
Multiples of 3: 3, 6, 9, 12
So, 12 is the LCD.

**Step 3**
Replace the unlike fractions with the like fractions.

\[
\begin{align*}
4\frac{1}{4} &= \frac{4 \times 3}{12} = \frac{12}{12} \\
-2\frac{2}{3} &= \frac{-2 \times 4}{12} = \frac{-8}{12}
\end{align*}
\]

**Step 5**
Subtract the fractions.

\[
\begin{align*}
4\frac{3}{12} - 2\frac{8}{12} &= \frac{12}{12} - \frac{8}{12} \\
&= 4\frac{3}{12} - 2\frac{8}{12}
\end{align*}
\]

**Step 6**
Subtract the whole numbers.

\[
\begin{align*}
4\frac{3}{12} - 2\frac{8}{12} &= \frac{12}{12} - \frac{8}{12} \\
&= 4 - 2 = 2
\end{align*}
\]

So, Wayne has $1\frac{7}{12}$ feet left.

Find the difference in simplest form.

1. \[5\frac{1}{4} - 1\frac{1}{2} = 3\frac{1}{4}\]
2. \[6\frac{1}{8} - 2\frac{1}{4} = 3\frac{1}{8}\]
3. \[5\frac{3}{10} - 1\frac{3}{5} = 3\frac{3}{10}\]
4. \[4\frac{1}{6} - 2\frac{2}{3} = 1\frac{1}{6}\]
Practice with Mixed Numbers

Larry made $2\frac{5}{6}$ pounds of baked ziti. He and his brother ate $1\frac{1}{3}$ pounds. How much was left over? Use fraction bars to find the answer.

Subtract. $2\frac{5}{6} - 1\frac{1}{3}$

Estimate: about $1\frac{1}{2}$ pound

$$2\frac{5}{6} - 1\frac{1}{3} = 1\frac{3}{6} \text{ or } 1\frac{1}{2} \text{ pounds}$$

Add or subtract. Write the answer in simplest form. Estimate to check.

1. $3\frac{13}{15} + 2\frac{1}{5} = 5\frac{4}{15}$
2. $1\frac{5}{12} + 2\frac{1}{6} = 3\frac{11}{12}$
3. $\frac{3}{4} - 3\frac{7}{8} = \frac{3}{8}$
4. $6\frac{2}{3} - 1\frac{10}{12} = 4\frac{1}{4}$

5. $\frac{3}{8} + 4\frac{7}{8} = 5\frac{1}{2}$
6. $\frac{4}{5} - 2\frac{2}{3} = -1\frac{1}{15}$
7. $\frac{7}{12} - 2\frac{1}{6} = -\frac{1}{4}$
8. $4\frac{2}{5} + 1\frac{1}{3} = 5\frac{3}{15}$

Algebra Find the value of $n$.

9. $3\frac{1}{2} + n = 5$
10. $n - 4\frac{1}{8} = 6\frac{1}{2}$

11. $4\frac{6}{7} - n = 2\frac{1}{7}$
12. $n + 11\frac{1}{6} = 15\frac{1}{3}$
Problem Solving Skill: Multistep Problems

Hank bought a piece of wood that was 8 feet long. He used \(1\frac{1}{4}\) feet for a shelf in his room and \(2\frac{1}{4}\) feet for a shelf in his sister’s room. Then he made a box using another \(3\frac{1}{4}\) feet. How much of the wood does he have left?

You can solve the problem by doing more than one operation. First add the \(1\frac{1}{4}\) feet for his shelf, the \(2\frac{1}{4}\) feet for his sister’s shelf, and the \(3\frac{1}{4}\) feet for his box.

\[
\begin{align*}
1\frac{1}{4} \\
2\frac{1}{4} \\
+3\frac{1}{4} \\
\hline
6\frac{3}{4}
\end{align*}
\]

Then you would subtract the total amount of \(6\frac{3}{4}\) feet from the 8 feet he bought. \(8 - 6\frac{3}{4} = 1\frac{1}{4}\)

So, Hank has \(1\frac{1}{4}\) feet of wood left.

Solve.

1. Ralph bought 12 feet of wood. He made four projects. The first one used \(3\frac{1}{2}\) feet, the second one used \(2\frac{1}{4}\) feet, the third one used \(2\frac{3}{4}\) feet, and the fourth one used \(1\frac{1}{4}\) feet. How much wood did he have left?

2. Nancy read every day for five days. She read 8 pages on Monday, 12 pages on Tuesday, 25 pages on Thursday, and 40 pages on Friday. If she read a total of 156 pages, how many pages did she read on Wednesday?

3. On Monday Charley drove 32 miles, on Tuesday 58 miles, on Wednesday 88 miles, and on Thursday 94 miles. His total for five days was 335 miles. How far did he drive on Friday?

4. Lacy was serving pizza at a party. She gave the first person \(\frac{1}{8}\) of the pizza, the second person \(\frac{3}{8}\), and the third person \(\frac{1}{4}\) of the pizza. How much of the pizza is left?
Multiply Fractions and Whole Numbers

Hector had 12 baseball cards. He gave \( \frac{2}{3} \) of them to his friend Ned. How many baseball cards did he give to Ned?

You can answer the question by multiplying \( \frac{2}{3} \times 12 \).
To multiply a fraction and a whole number you can use a model:

Step 1  Draw 12 rectangles to show the cards.

Step 2  The denominator of the fraction \( \frac{2}{3} \) is 3. This means there are 3 equal parts, so divide the rectangles into 3 equal groups.

Step 3  The numerator of the fraction \( \frac{2}{3} \) is 2. This means there are 2 parts given, so shade 2 of the groups.

Step 4  Count the shaded rectangles, or cards. There are 8 cards.

So, \( \frac{2}{3} \times 12 = 8 \).

Write the number sentence each model represents.

1. \[ \frac{4}{9} \times 27 = \]
2. \[ \frac{1}{6} \times 12 = \]
3. \[ \frac{3}{5} \times 20 = \]

Draw a picture to help you multiply. Find the product.

4. \( \frac{4}{9} \times 27 = \)
5. \( \frac{1}{6} \times 12 = \)
6. \( \frac{3}{5} \times 20 = \)
Multiply a Fraction by a Fraction

Multiply.  \( \frac{3}{4} \times \frac{3}{5} \)

To multiply fractions you can use a rectangle model. Follow these guidelines:

- Draw a rectangle, and divide the rectangle into 5 equal columns. This is for the denominator of \( \frac{3}{5} \).
- Shade 3 of the columns. This is for the numerator of \( \frac{3}{5} \).
- Divide the rectangle into 4 equal rows. This is for the denominator of \( \frac{3}{4} \).
- Shade 3 of the rows with diagonal lines. This is for the numerator of \( \frac{3}{4} \).
- Count how many pieces the rectangle is divided into. There are 20 pieces. This is the new denominator.
- Count how many pieces have overlapping lines and shading. There are 9. This is the new numerator.

So, \( \frac{3}{4} \times \frac{3}{5} = \frac{9}{20} \).

Divide and shade a rectangle model to find the product.

1. \( \frac{1}{3} \times \frac{5}{6} = \) __________
2. \( \frac{5}{8} \times \frac{3}{4} = \) __________
3. \( \frac{1}{4} \times \frac{3}{8} = \) __________

4. \( \frac{2}{5} \times \frac{1}{3} = \) __________
5. \( \frac{1}{2} \times \frac{7}{8} = \) __________
6. \( \frac{5}{6} \times \frac{3}{4} = \) __________

7. \( \frac{1}{4} \times \frac{5}{6} = \) __________
8. \( \frac{2}{3} \times \frac{1}{4} = \) __________
9. \( \frac{2}{7} \times \frac{3}{4} = \) __________

10. \( \frac{3}{5} \times \frac{3}{5} = \) __________
11. \( \frac{4}{5} \times \frac{1}{2} = \) __________
12. \( \frac{5}{9} \times \frac{1}{2} = \) __________
Multiply Fractions and Mixed Numbers

Multiply. $\frac{2}{3} \times 2\frac{1}{4}

You can find the product by using the Distributive Property. The Distributive Property allows you to break apart numbers to multiply.

To multiply a fraction and a mixed number, break apart the mixed number.

$$\frac{2}{3} \times 2\frac{1}{4} = \frac{2}{3} \times \left(2 + \frac{1}{4}\right) \quad \text{Break apart the mixed number.}$$

$$= \left(\frac{2}{3} \times 2\right) + \left(\frac{2}{3} \times \frac{1}{4}\right) \quad \text{Multiply each part.}$$

$$= \frac{4}{3} + \frac{2}{12}$$

$$= \frac{16}{12} + \frac{2}{12} \quad \text{Find the LCD and rename the fractions.}$$

$$= \frac{18}{12} = \frac{1\frac{1}{2}}{2} \quad \text{Add the products. Simplify the sum.}$$

So, $\frac{2}{3} \times 2\frac{1}{4} = 1\frac{1}{2}$.

Multiply. Write the answer in simplest form.

1. $\frac{1}{3} \times 3\frac{1}{5} = \underline{\phantom{0}}$

2. $\frac{1}{2} \times 2\frac{3}{4} = \underline{\phantom{0}}$

3. $\frac{1}{6} \times 3\frac{2}{3} = \underline{\phantom{0}}$

4. $\frac{1}{4} \times 2\frac{5}{6} = \underline{\phantom{0}}$

5. $\frac{1}{3} \times 3\frac{1}{2} = \underline{\phantom{0}}$

6. $\frac{1}{8} \times 4\frac{1}{4} = \underline{\phantom{0}}$

7. $\frac{3}{8} \times 1\frac{1}{4} = \underline{\phantom{0}}$

8. $\frac{4}{5} \times 2\frac{1}{2} = \underline{\phantom{0}}$
Multiply with Mixed Numbers

Multiply. $\frac{2}{3} \times \frac{1}{2}$

To multiply two mixed numbers, follow the same steps you use to multiply a fraction and a mixed number.

**Step 1**

Write each mixed number as a fraction.

\[
\frac{2}{3} = \frac{(3 \times 1) + 2}{3} = \frac{5}{3}
\]

\[
\frac{1}{2} = \frac{(2 \times 1) + 1}{2} = \frac{3}{2}
\]

**Step 2**

Multiply the fractions, or cancel the 3 in the numerator and denominator.

\[
\frac{5 \times 3}{3 \times 2} = \frac{15}{6}
\]

\[
\frac{5}{2} = \frac{2 \frac{3}{6}}{2 \frac{1}{6}}
\]

or cancel the 3 in the numerator and denominator.

\[
\frac{5 \times \frac{1}{3}}{3 \times 2} = \frac{5}{2}
\]

**Step 3**

Write the product as a mixed number in simplest form.

\[
\frac{15}{6} = \frac{2 \frac{3}{6}}{2 \frac{1}{6}}
\]

or

\[
\frac{5}{2} = \frac{2 \frac{1}{2}}{2 \frac{1}{2}}
\]

Multiply. Write the answer in simplest form.

1. $\frac{2}{2} \times \frac{1}{5} = \quad \quad 2. \quad \frac{1}{3} \times \frac{1}{2} = \quad \quad$

3. $\frac{1}{2} \times \frac{1}{4} = \quad \quad 4. \quad \frac{3}{4} \times \frac{3}{2} = \quad \quad$

5. $\frac{6}{2} \times \frac{3}{5} = \quad \quad 6. \quad \frac{2}{3} \times \frac{2}{3} = \quad \quad$

7. $\frac{1}{5} \times \frac{1}{2} = \quad \quad 8. \quad \frac{1}{2} \times \frac{3}{5} = \quad \quad$
Problem Solving Skill

Sequence and Prioritize Information

The Perez family planned an evening event that includes a snack, dinner, dessert, and game time.

There are 6 hours planned for the evening. $\frac{1}{3}$ of the evening’s time will be devoted to dinner. $\frac{1}{6}$ of the time will be spent on having a snack. How many hours will be spent on playing games and dessert?

Sequencing the information may help you solve this problem. Start with events for which you have some information.

<table>
<thead>
<tr>
<th>Event</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dinner</td>
<td>$6\text{ hours total } \times \frac{1}{3} = 2 \text{ hours for dinner}$</td>
</tr>
<tr>
<td>Snack</td>
<td>$6\text{ hours total } \times \frac{1}{6} = 1 \text{ hour for snacks}$</td>
</tr>
</tbody>
</table>

Now subtract the snack and dinner time to find how much time can be devoted to games and dessert.

<table>
<thead>
<tr>
<th>Event</th>
<th>Time</th>
</tr>
</thead>
</table>
| Games and Dessert | Total time spent on dinner and snacks  
                | $= 3 \text{ hours}$                                       |
|                | $6\text{ hours total } - 3 \text{ hours for dinner and snacks}$  
                | $= 3 \text{ hours left for games and dessert}$ |

Sequence the information by starting with the information you know. Then solve the problem.

1. John drives a total of 350 miles a day. He makes 3 stops. He drives 150 miles to his first stop. From the second stop to the third stop, he drives 75 miles. How many miles does he drive from the first stop to the second stop?

2. Mary spent $45.00 altogether at the store. She bought some food for $32.75 and some school supplies. How much did she spend on school supplies?
Divide Fractions
You can use pictures to model division of fractions.

\[ \frac{5}{3} \div \frac{1}{3} \quad \text{and} \quad \frac{4}{5} \div \frac{1}{10} \]

**Step 1:** Draw 5 whole circles and shade all 5.

**Step 2:** Divide each circle into thirds.

**Step 3:** Count the number of shaded thirds.
There are 15 thirds in 5. So, \( 5 \div \frac{1}{3} = 15 \).

**Step 1:** Draw one whole rectangle and shade four fifths of it.

**Step 2:** Divide the rectangle into tenths.

**Step 3:** Count the number of shaded tenths.
There are 8 tenths in \( \frac{4}{5} \). So, \( \frac{4}{5} \div \frac{1}{10} = 8 \).

Draw a model for the division problem and find the quotient.

1. \( \frac{2}{3} \div \frac{1}{9} = \) ______

2. \( 2 \div \frac{1}{5} = \) ______

3. \( \frac{3}{4} \div \frac{1}{8} = \) ______

4. \( 3 \div \frac{1}{4} = \) ______

5. \( \frac{1}{2} \div \frac{1}{8} = \) ______

6. \( \frac{1}{3} \div \frac{1}{6} = \) ______
Reciprocals

Reciprocals are two fractions that have a product of 1.

**Fractions:**
To find the reciprocal of a fraction, switch the numerator and denominator.

The reciprocal of \( \frac{3}{8} \) is \( \frac{8}{3} \).

\[
\frac{3}{8} \times \frac{8}{3} = \frac{24}{24} = 1
\]

**Whole Numbers:**
To find the reciprocal of a whole number, first write it as a fraction. Then switch the numerator and denominator.

The reciprocal of \( \frac{7}{1} \) is \( \frac{1}{7} \).

\[
7 = \frac{7}{1}.
\]

**Mixed Numbers:**
To find the reciprocal of a mixed number, first write it as a fraction. Then switch the numerator and denominator.

The reciprocal of \( \frac{17}{3} \) is \( \frac{3}{17} \).

\[
5\frac{2}{3} = \frac{17}{3}
\]

Are the two numbers reciprocals? Write yes or no.

1. \( \frac{1}{9} \) and 19
2. \( \frac{3}{10} \) and \( \frac{10}{3} \)
3. \( 1\frac{3}{5} \) and \( \frac{8}{5} \)
4. 5 and \( \frac{1}{5} \)

5. \( \frac{5}{13} \) and \( 2\frac{3}{5} \)
6. \( \frac{1}{10} \) and \( \frac{1}{10} \)
7. \( 2\frac{1}{4} \) and \( \frac{4}{9} \)
8. \( \frac{7}{12} \) and \( \frac{12}{7} \)

Write the reciprocal of each number.

9. \( \frac{1}{7} \)
10. \( \frac{5}{12} \)
11. 6
12. \( 3\frac{5}{9} \)
13. \( \frac{6}{5} \)

14. \( \frac{2}{11} \)
15. 11
16. \( 1\frac{3}{8} \)
17. \( \frac{1}{2} \)
18. 100
Divide Whole Numbers by Fractions

Beth is working on a science project. She needs pieces of wire that are \( \frac{2}{3} \) yd long for the project. She bought a piece of wire that is 6 yd long at the hardware store.

How many \( \frac{2}{3} \) pieces can she cut from this 6-yd piece?

**Step 1:** Write a division sentence to find this amount.

\[
\frac{6}{1} \div \frac{2}{3}
\]

Think: Write 6 as \( \frac{6}{1} \).

**Step 2:** Use the reciprocal of the divisor to write a multiplication problem.

\[
\frac{6}{1} \times \frac{3}{2}
\]

Think: The reciprocal of \( \frac{2}{3} \) is \( \frac{3}{2} \).

**Step 3:** Multiply.

\[
\frac{6}{1} \times \frac{3}{2} = \frac{18}{2} = 9
\]

So, Beth can cut 9 pieces of wire.

---

Use the reciprocal to write a multiplication problem. Solve the problem. Write the answer in simplest form.

1. \( 3 \div \frac{1}{8} \)  
   \[
   \frac{3}{1} \times \frac{8}{1} = 24
   \]

2. \( 5 \div \frac{1}{2} \)  

3. \( 10 \div \frac{2}{3} \)

4. \( 27 \div \frac{3}{5} \)

5. \( 12 \div \frac{4}{5} \)

6. \( 8 \div \frac{3}{4} \)

7. \( 18 \div \frac{3}{8} \)

8. \( 7 \div \frac{4}{5} \)

9. \( 6 \div \frac{3}{4} \)

10. \( 16 \div \frac{4}{5} \)

11. \( 9 \div \frac{6}{7} \)

12. \( 2 \div \frac{3}{10} \)

13. \( 9 \div \frac{3}{8} \)

14. \( 9 \div \frac{1}{5} \)

15. \( 6 \div \frac{3}{20} \)

16. \( 20 \div \frac{4}{5} \)
Divide Fractions

Connie is working on a craft project. She needs \( \frac{3}{8} \)-yd pieces of ribbon for the project. She bought a \( \frac{3}{4} \)-yd piece of ribbon at the craft store.

How many \( \frac{3}{8} \)-yd pieces can she cut from \( \frac{3}{4} \)-yd piece?

**Step 1:** Write a division sentence to find this amount.

\[
\frac{3}{4} \div \frac{3}{8}
\]

**Step 2:** Use the reciprocal of the divisor to write a multiplication problem.

\[
\frac{3}{4} \times \frac{8}{3} \quad \text{Think: The reciprocal of} \quad \frac{3}{8} \quad \text{is} \quad \frac{8}{3}.
\]

**Step 3:** Multiply.

\[
\frac{3}{4} \times \frac{8}{3} = \frac{24}{12} = 2
\]

So, Connie can cut 2 pieces of ribbon.

---

Use the reciprocal to write a multiplication problem. Solve the problem. Write the answer in simplest form.

1. \( \frac{3}{8} \div 24 \)  
2. \( \frac{5}{9} \div \frac{2}{3} \)  
3. \( \frac{4}{5} \div \frac{2}{3} \)  
4. \( \frac{5}{12} \div \frac{5}{8} \)

---

5. \( \frac{5}{6} \div \frac{1}{3} \)  
6. \( \frac{5}{8} \div \frac{3}{4} \)  
7. \( \frac{4}{5} \div 6 \)  
8. \( 1\frac{1}{15} \div 7 \)

---

9. \( 2\frac{1}{4} \div \frac{1}{3} \)  
10. \( 1\frac{1}{4} \div 2\frac{1}{3} \)  
11. \( \frac{1}{3} \div \frac{1}{2} \)  
12. \( 1\frac{1}{3} \div 1\frac{1}{2} \)

---

13. \( \frac{1}{2} \div \frac{1}{4} \)  
14. \( \frac{3}{4} \div 1\frac{1}{4} \)  
15. \( \frac{5}{6} \div \frac{1}{3} \)  
16. \( 1\frac{2}{3} \div \frac{1}{3} \)
Problem Solving Strategy

Solve a Simpler Problem

The bank gave Jim a loan of $4,000. This is $\frac{1}{8}$ of the amount they gave him last year. How much did the bank loan Jim last year?

You can solve a more difficult problem by first solving a simpler one.

**Step 1:** If you can, change the numbers so that they are easier to work with.

Let 4 represent 4,000.

**Step 2:** Write the problem, using the new number.

$$4 \div \frac{1}{8} = \frac{4}{1} \times \frac{8}{1}$$

**Think:** The reciprocal of $\frac{1}{8}$ is $\frac{8}{1}$.

**Step 3:** Solve the problem, using the new number.

$$\frac{4}{1} \times \frac{8}{1} = 32$$

**Step 4:** Adjust the answer, using the original number.

Multiply the answer by 1,000 to adjust.

So, $32 \times 1,000 = 32,000$.

So, the bank loaned Jim $32,000 last year.

---

Use a simpler problem to solve. Then adjust your answer.

1. Charles spent $600 on a new bike. This was $\frac{2}{3}$ of his savings. How much money was in his savings?

2. The distance from Barbara’s house to Raymond’s house is 3,200 miles. You can travel $\frac{3}{4}$ of the distance by highway. How many miles cannot be traveled by highway?
Integers

Integers are whole numbers and their opposites.

The positive integers are to the right of 0. The negative integers are to the left of 0. 0 is neither positive nor negative.

How far from 0 is $+5$ and in what direction? ________________

How far from 0 is $-5$ and in what direction? ________________

$+5$ and $-5$ are opposites. They are the same distance from 0 on a number line, but in opposite directions. Some other opposites are $+1$ and $-1$, $+6$ and $-6$.

The distance a number is from 0 is referred to as its absolute value. $+5$ and $-5$ are both 5 units from 0. So, $| -5 |$ and $| +5 |$ both equal 5.

How far from 0 is $-8$? 8 units
So, $| -8 | = 8$ because it is 8 units from 0.

Name the integer which describes each situation below.

1. an increase in price of $30.00  
2. 10 minutes before school starts  
3. 4 feet above ground

Write the opposite of each integer.

4. $+6$ ______ 5. $-28$ ______ 6. $+1,489$ ______ 7. $-2,000$ ______

Name each integer’s absolute value.

8. $| +34 |$ ______ 9. $| -30 |$ ______ 10. $| -235 |$ ______
Compare and Order Integers

Integers increase as you move right on a number line and decrease as you move left.

Compare \(-7\) to \(-8\). Use \(<\), \(>\) or \(=\).

\[
\begin{array}{cccccccccccc}
-10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & +1 & +2 & +3 & +4 & +5 & +6 & +7 & +8 & +9 & +10
\end{array}
\]

The numbers increase as you move right, and \(-7\) is to the right of \(-8\). So, \(-7 > -8\).

Order \(+5\), \(-5\), \(-3\), and \(+7\) from least to greatest.

\[
\begin{array}{cccccccccccc}
-10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & +1 & +2 & +3 & +4 & +5 & +6 & +7 & +8 & +9 & +10
\end{array}
\]

Look at the number line. Since the numbers increase as you move right on the number line, the order from least to greatest is \(-5\), \(-3\), \(+5\), \(+7\).

Name the integer that is 1 less than \(-8\).

\[
\begin{array}{cccccccccccc}
-10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & +1 & +2 & +3 & +4 & +5 & +6 & +7 & +8 & +9 & +10
\end{array}
\]

1 less means a decrease, and decreasing amounts move left on a number line. So, 1 to the left of \(-8\) is \(-9\).

---

Compare. Write \(<\), \(>\), or \(=\) for each \(\bigcirc\).

1. \(+6 \bigcirc +6\) 2. \(-7 \bigcirc -2\) 3. \(+3 \bigcirc -1\) 4. \(-10 \bigcirc +8\)

Order each set of integers from greatest to least.

5. \(-9\), \(+1\), \(0\), \(-4\) 6. \(+2\), \(-5\), \(+6\), \(-4\) 7. \(+10\), \(-10\), \(+1\), \(-1\) 8. \(-4\), \(+2\), \(-5\), \(-1\)

Name the integer that is one less than the given integer.

9. \(+6\) 10. \(-4\) 11. \(0\) 12. \(-21\) 13. \(+25\)

Name the integer that is one more than the given integer.

14. \(-10\) 15. \(+4\) 16. \(-3\) 17. \(+32\) 18. \(-1\)
Add Integers

This pan balance “weighs” positive and negative numbers. Negative numbers go on the left of the balance and positive numbers go on the right.

Find $-11 + 8$.

The scale will tip to the left side, because it is $-3$ “heavier.”

Find $-2 + 7$.

The scale will tip to the right side, because it is $+5$ “heavier.”

Find $-1 + -3$.

Both $-1$ and $-3$ go on the left side. The scale will tip to the left side, because it is $-4$ “heavier.”

Find how much “heavier” the lower side is.

1. $2 + -7$
   - Find how much “heavier” the lower side is.
   - Solve.

2. $6 + -10$

3. $8 + -9$

4. $-10 + -10$

5. $-10 + 2$

6. $3 + 9$

Solve.

7. $+7 + -4$
   - $7 + -4$

8. $+10 + -4$
   - $10 + -4$

9. $-6 + 0$
   - $-6 + 0$

10. $-5 + -4$
    - $-5 + -4$

11. $-9 + +3$
12. $+9 + -1$
13. $+5 + -3 + -2$
14. $-3 + +5$

Reasoning Without adding, tell whether the sum will be negative, positive, or zero.

15. $-18 + +25$
16. $+9 + -20$
17. $+427 + -427$
18. $+75 + -19$
Subtract Integers

You can use drawings to subtract integers.

Use circles with + signs to represent positive integers and circles with – signs to represent negative integers.

Find \(+8 - +6\).

**Step 1**
First, make a drawing of \(+8\).

\[ + + + + + + + + \]

**Step 2**
To subtract \(+6\), take away 6 of the + circles.

\[ + + + + + + \rightarrow + + + + + + + \]

**Step 3**
The number of circles left represents the difference.

So, \(+8 - +6 = +2\).

Find \(-5 - +3\).

**Step 1**
First, make a drawing of \(-5\).

\[ - - - - - \]

**Step 2**
You cannot subtract \(+3\) until you add positive circles.

\[ - - - - - \]

**Step 3**
To subtract \(+3\), take away 3 of the + circles.

\[ - - - \rightarrow + + + \]

**Step 4**
The number of circles left represents the difference.

So, \(-5 - +3 = -8\).

Solve.

1. \(+5 - +3\)  
2. \(-6 - +2\)  
3. \(+7 - +3\)  
4. \(-9 - +2\)

5. \(-8 - +1\)  
6. \(+10 - +4\)  
7. \(-4 - +4\)  
8. \(-7 - +6\)

9. \(-5 - +4\)  
10. \(+8 - +2\)  
11. \(-7 - +6\)  
12. \(-9 - +3\)
**Subtract Integers**

When you subtract a positive integer from a negative integer, you add the opposite. Replace the negative integer with its opposite, and change the operation to addition.

**Example:** Find $-6 - 2$.

**Step 1** Find the opposite of the number being subtracted. The opposite of $+2$ is $-2$.

**Step 2** Add the opposite. Change the operation to addition and replace the number being subtracted with its opposite. $-6 - 2 = -6 + -2$

**Step 3** Add the negative numbers.

So, $-6 + -2 = -8$.

---

**Solve.**

1. $-5 - 3$
2. $-4 - 7$
3. $-2 - 8$
4. $-7 - 1$

5. $-2 - 1$
6. $-4 - 6$
7. $-7 - 3$
8. $0 - 8$

9. $-9 - 2$
10. $-5 - 5$
11. $-2 - 4$
12. $-5 - 1$

---

**Algebra Complete.**

13. $-2 + 6 = -2 + [\square]$
14. $-7 + 7 = -7 + [\square]$
15. $-3 + 9 = -3 + [\square]$
16. $-5 + 5 = -5 + [\square]$

**Compare. Write $<$, $>$, or $=$ in each $\bigcirc$.**

17. $-5 + 6 \bigcirc -2 + 6$
18. $2 + 8 \bigcirc 5 + -7$
19. $-3 + 7 \bigcirc -3 + -7$
20. $-5 + 2 \bigcirc -6 + 1$
Problem Solving Strategy

Draw a Diagram

Draw a diagram to solve.

**PROBLEM:** Erik and his friends are practicing scuba diving in a 20-foot-deep, 30-foot-long pool for class. First, they had to go down 15 feet. Then they had to go down 2 more feet to practice clearing the water out of their masks. Then they went up 9 feet and back down 10 feet. At what depth are they now?

When solving problems involving integers, first look for key words to determine the positive and negative numbers. A few key words are listed.

- **negative (−)**
- **positive (+)**
- **down**
- **up**
- **below**
- **above**
- **drop**
- **raise**

The key words in the problem are **up** (+) and **down** (−). Use the key words to step out the problem.

**Step 1** down 15 feet (−15)

**Step 2** down 2 more feet (−2)

**Step 3** up 9 feet (+9)

**Step 4** down 5 feet (−5)

Making or using a diagram will help you visualize the steps to solve the problem. Look at the diagram.

The last step shows that Erik and his friends will go from −8 feet down 5 more (−5) feet. The students are now at 13 feet below the surface, or −13 feet.

Draw a diagram to solve.

1. The next day a new set of divers were practicing in the pool. They began by diving 19 feet. Then they rose 9 feet, went back down 7 feet and up 5 feet. Where are they now?

2. Jan went swimming. She dove 15 feet, came up 8 feet, went down 1 foot, and came back up 5 more feet. Where is she now?
Bill put his collection of pennies in $0.50 rolls. Every two rolls held $1. He made a table to show the relationship between number of dollars and number of rolls of pennies.

Bill wrote the data as ordered pairs: (1,2), (2,4), (3,6), and (4,8). Then he graphed the points and drew a line to connect them.

The ordered pair (2,4) means that Bill has $2 if he has 4 rolls of pennies.

Write the ordered pairs. Then graph the ordered pairs.

1. Input, x  
   Output, y  
   \[
   \begin{array}{c|c|c|c|c}
   x & 1 & 2 & 3 & 4 \\
   \hline
   y & 4 & 8 & 12 & 16 \\
   \end{array}
   \]

2. Input, x  
   Output, y  
   \[
   \begin{array}{c|c|c|c|c}
   x & 8 & 10 & 12 & 14 \\
   \hline
   y & 4 & 5 & 6 & 7 \\
   \end{array}
   \]

3. Input, x  
   Output, y  
   \[
   \begin{array}{c|c|c|c}
   x & 2 & 4 & 6 & 8 \\
   \hline
   y & 3 & 5 & 7 & 9 \\
   \end{array}
   \]

4. Input, x  
   Output, y  
   \[
   \begin{array}{c|c|c|c|c}
   x & 8 & 7 & 6 & 5 \\
   \hline
   y & 6 & 5 & 4 & 3 \\
   \end{array}
   \]

5. In the problem with Bill’s pennies, what does the ordered pair (3,6) mean?

6. In the problem with Bill’s pennies, what would be the next ordered pair?

7. How did you decide the answer for problem 6?
Graph Integers on the Coordinate Plane

A coordinate plane is formed by a horizontal number line (x-axis) and a vertical number line (y-axis), which intersect. The point at which the two lines intersect is named by the ordered pair (0,0) and is called the origin. The numbers in the ordered pair are called coordinates.

To plot ordered pairs on a coordinate plane, begin at the origin. Positive numbers are to the right and above (0,0). Negative numbers are to the left and below (0,0).

Write the ordered pair described. Then plot and label the point on the coordinate plane.

1. Start at the origin. Move right 5 units and up 3 units. 
2. Start at the origin. Move left 4 units and up 1 unit. 
3. Start at the origin. Move right 2 units and down 3 units. 
4. Start at the origin. Move left 1 unit and down 3 units.

Identify the ordered pair for each point on the coordinate plane above.

5. Point A 
6. Point B 
7. Point C 
8. Point D 
9. Point E 
10. Point F
Use an Equation to Graph

Sally earns $3 for each hour that she baby-sits. She can graph how much she earns according to the number of hours she baby-sits.

Sally can show this relationship with x and y values in a function table.

<table>
<thead>
<tr>
<th>Hours, x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earns, y</td>
<td>$3</td>
<td>$6</td>
<td>$9</td>
<td>$12</td>
<td>$15</td>
</tr>
</tbody>
</table>

This table shows the amount of money that Sally can earn for the hours she baby-sits. There is a pattern. Find the rule and write the equation that shows the relationship between x and y.

Rule: Multiply x by 3. Equation: \( y = 3x \)

Sally knows that for any number of hours, she earns three times that number.

She can find out exactly how much she will make by substituting the number of hours (x) and multiplying it by 3 to get y, how much she earns.

Use a rule to complete the table. Then write the equation.

1. Table, x | 1 | 2 | 3 | 4 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Table legs, y</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>__</td>
</tr>
</tbody>
</table>

2. Fingers, x | 5 | 10 | 15 | 20 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Hands, y</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>__</td>
</tr>
</tbody>
</table>

3. Input, x | 2 | 3 | 4 | 7 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output, y</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>__</td>
</tr>
</tbody>
</table>

4. child, x | 1 | 2 | 3 | 4 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>eyes, y</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>__</td>
</tr>
</tbody>
</table>

Use each equation to make a table, write 4 ordered pairs, and then make a graph.

5. \( y = x + 4 \)

<table>
<thead>
<tr>
<th>x</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. \( y = x \div 2 \)

<table>
<thead>
<tr>
<th>x</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. \( y = x + -4 \)

<table>
<thead>
<tr>
<th>x</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Problem Solving Skill**

**Relevant or Irrelevant Information**

Jonathan gave a map to his visiting cousin so she would be able to find the places she needs. She is looking for the Pet Store and knows that its x-coordinate is the same as the Food Store’s x-coordinate. The Snack Shop is north of the Cinema. Jonathan said that the Pet Store is 4 blocks south of the Toy Store. Can you help her find the Pet Store?

**Step 1**

Decide what you are trying to find.

- the coordinates of the Pet Store

**Step 2**

Read each fact and decide whether it is relevant or irrelevant to solving the problem.

- The Pet Store has the same x-coordinate as the Food Store. *relevant*
- The Snack Shop is irrelevant north of the Cinema.
- The Pet Store is 4 blocks relevant south of the Toy Store.

**Step 3**

Use the relevant information to solve the problem.

- The Food Store’s x-coordinate is $+3$.
- The Toy Store’s y-coordinate is $+5$.
- So the Pet Store is at $(+3, +1)$.

---

1. A group of 72 students visited the science center. One third of them visited the planetarium. One half of that number went to the weather exhibit. The remaining students visited the electricity exhibit. Most of the students liked the science center. How many students saw the electricity exhibit?

2. Which information is relevant to this problem?
**Lines and Angles**

In geometry, objects have special names.

You can make lines by connecting any two points. Lines go on forever. You show this by putting arrows at the ends of the line.

Lines that cross at one point are **intersecting**.

Lines that intersect to form four right angles are **perpendicular**.

Lines in a plane that never intersect and are the same distance from each other are **parallel**.

A **line** is a straight path in a plane. It has no ends. It can be named by any two points on the line.

A **line segment** is part of a line. It is the shortest distance between two points on a line.

---

**Draw and label each object.**

1. lines $AB$ and $CD$ parallel to each other

2. line segment $KL$

3. line $FG$

4. lines $EF$ and $GH$ intersecting at point $A$

5. lines $NO$ and $QR$ perpendicular to each other

6. lines $HI$ and $JK$ parallel to each other
Measure and Draw Angles

You can use a protractor to measure the angle at the right. A protractor is a tool for measuring the size of the opening of an angle. The unit used to measure an angle is a degree.

A protractor has a center point at the bottom where two lines form right angles. To the right of this is the 0° mark. To the left is the 180° mark.

**Step 1**
Place the protractor on the angle so that the center point lines up with the vertex and the horizontal line on the protractor lines up with ray $EF$.

**Step 2**
To measure the angle, place a pencil on top of the other ray of the angle.

Read the number of degrees the pencil is pointing to.

So, the measure of $\angle DEF$ is 50°.

Use a protractor to measure and classify the angle.

1. 
2. 
3. 

4. 
5. 
6.
Angles and Polygons

How can you remember polygons and their angles? One way is to learn the meanings of the words that describe each shape. Remembering other words that use the same roots can also help you remember the figures.

**triangle**  
*tri*- means 3
*tricycle*— a 3-wheeled bicycle

**quadrilateral**  
*quad*- means 4
*quadruplets* — 4 babies born at once to the same mother

**pentagon**  
*pent*- means 5
*the Pentagon* — a 5-sided building in Washington, D.C.

**hexagon**  
*hex*- means 6
*hex sign* — Pennsylvania Dutch art that uses 6-sided figures drawn inside a circle

**octagon**  
*oct*- means 8
*octopus* — an animal with 8 legs
*October* — used to be the 8th month on the calendar

**polygon**  
*poly*- means many
*polyhedra* — a term used for the many-sided shapes of crystals

Name each polygon.

1. 2. 3. 4. 5. 6. 7. 8.

Many quadrilateral shapes have their own meanings as well. Look in the dictionary and find these meanings. Find another word that can help you remember the meaning. Then draw the shape.

9. trapezoid  10. parallelogram  11. rectangle  12. rhombus
Circles

You need a centimeter ruler and a compass to construct a circle with a radius of 2 cm.

A radius is a line segment that connects the center with a point on the circle.

A diameter is a chord that passes through the center of a circle.

- Draw a point at the center of the circle.
- Start at the point. Use a centimeter ruler to draw a line segment 2 cm long. This is the radius.
- Place the point of the compass on the center point. Place the pencil point on the other end of the radius.
- Hold the compass point still. Turn the compass around on the point to make a complete circle.

Use a centimeter ruler and a compass to construct each circle.

1. radius = 1 cm  
2. radius = 1.5 cm  
3. radius = 2 cm

4. diameter = 1.0 cm  
5. diameter = 2.4 cm  
6. diameter = 5.0 cm
**Congruent and Similar Figures**

Two figures are similar if they have the same shape.

Two figures are congruent if their matching sides and angles are the same.

To determine if triangles $ABC$ and $DEF$ are congruent:

- Measure the sizes of the matching angles to see if they are equal.
- Measure the lengths of the matching sides to see if they are equal.

The matching sides are equal, and the matching angles are equal. So, the two triangles are congruent.

Find one pair of similar figures and four pairs of congruent figures.

A. B. C. D. E. F. G. H. I. J.
**Symmetric Figures**

A figure has line symmetry when it can be folded on a line so that its two parts match. The two halves of the pentagon are congruent.

Trace the pentagon. Fold it in half along the dotted line. The left half is congruent to the right half. A figure can have more than one line of symmetry. Find all the lines of symmetry for the pentagon.

The pentagon has five lines of symmetry in all.

Draw the lines of symmetry. How many lines of symmetry does each figure have?

1. __________ 2. __________ 3. __________

4. __________ 5. __________ 6. __________
Problem Solving Strategy

Find a Pattern

Leonardo Fibonacci was one of the most talented mathematicians of the Middle Ages. One of his hobbies was studying number patterns. One of his most famous patterns is shown below. What is the next term in the pattern?

1, 1, 2, 3, 5, 8

Step 1  What does the problem ask? It asks what the next term in the number pattern is.

Step 2  Find a pattern. The next term is the sum of the two previous terms.

\[1 + 1 = 2, \quad 1 + 2 = 3, \quad 2 + 3 = 5, \quad 3 + 5 = 8\]

Step 3  Use this information to solve the problem.

\[5 + 8 = 13\]  13 is the next term in the pattern.

Find a pattern to solve.

1. What is the next shape in this pattern?

   □ □ □ □ □ □ □

   ______________________

2. When Fred's number is 1, Ann's number is 3. When Fred's number is 2, Ann's number is 5. If Fred's is 6, what is Ann's number?

   ______________________

3. Write a rule for the pattern described in Problem 2.

   ______________________

   ______________________

   ______________________

4. Alex read 45 pages on Sunday, 90 pages on Monday and 135 pages on Tuesday. If he continues this pattern, how many pages will he read on Friday?

   ______________________
**Triangles**

Triangles are polygons with 3 sides and 3 angles. One method of classifying triangles is by the lengths of their sides.

To classify a triangle using this method, you need to know the lengths of its sides.

3 congruent sides = **equilateral** triangle

2 congruent sides = **isosceles** triangle

0 congruent sides = **scalene** triangle

Each side is a different length, so this is a scalene triangle.

List the number of congruent sides. Then name each triangle. Write **isosceles**, **scalene**, or **equilateral**.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9.
Quadrilaterals

Polygons that have 4 sides and 4 angles are **quadrilaterals**. Quadrilaterals can be classified by looking at the number of parallel sides, the lengths of their sides, and the measures of their angles.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Trapezoid</th>
<th>Parallelogram</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of parallel sides</td>
<td>1 pair</td>
<td>2 pairs</td>
<td>2 pairs</td>
<td>2 pairs</td>
<td>2 pairs</td>
</tr>
<tr>
<td>Number of congruent sides</td>
<td>0 pair</td>
<td>2 pairs</td>
<td>2 pairs</td>
<td>all 4 sides</td>
<td>all 4 sides</td>
</tr>
<tr>
<td>Number of congruent angles</td>
<td>0 pair</td>
<td>2 pairs</td>
<td>all 4 angles</td>
<td>2 pairs</td>
<td>all 4 angles</td>
</tr>
</tbody>
</table>

To classify the quadrilateral at the right, identify the following characteristics.

Number of parallel sides: 2 pairs
Number of congruent sides: all 4 sides
Number of congruent angles: 2 pairs

So, the figure is a **rhombus**.

Classify each quadrilateral. Write quadrilateral, trapezoid, parallelogram, rectangle, rhombus, or square.

1. [Diagram]
2. [Diagram]
3. [Diagram]
4. [Diagram]
5. [Diagram]
6. [Diagram]
Algebra: Transformations

When you move a figure, it is called a rigid transformation. A translation is one type of transformation.

When you translate, or slide, a figure on a coordinate plane, the coordinates change. The figure may move up or down, left or right, or both. Here are three examples of translations.

**3 spaces to the right**

- New ordered pairs: (2,1) to (5,1),
  (4,1) to (7,1),
  (2,4) to (5,4),
  (4,4) to (7,4)

**2 spaces up**

- New ordered pairs: (2,1) to (2,3),
  (4,1) to (4,3),
  (2,4) to (2,6),
  (4,4) to (4,6)

**3 spaces to the right and 2 spaces up**

- New ordered pairs: (2,1) to (5,3),
  (4,1) to (7,3),
  (2,4) to (5,6),
  (4,4) to (7,6)

Translate each figure. Draw the new figure with its coordinates. Name the new ordered pairs.

1. Translate the figure 5 spaces to the right and 4 spaces up.
2. Translate the figure 3 spaces to the right and 4 spaces down.
3. Translate the figure 4 spaces to the left and 4 spaces down.
Solid Figures

A **prism** is a solid figure that has two congruent faces called **bases**. A prism is named by the polygons that form its bases.

The prism at the right is a hexagonal prism.
- The faces of this solid figure are rectangles.
- The bases of this solid figure are hexagons.

A **pyramid** is a solid figure with one base that is a polygon and three or more faces that are triangles with a common vertex. A pyramid is named by the polygon that forms its base.

This is a hexagonal pyramid.
- The faces of this solid figure are triangles.
- The base of this solid figure is a hexagon.

Classify the solid figure. Then write the number of faces, vertices, and edges.

1. I have a base with 5 equal sides. My faces are 5 triangles.

2. All 6 of my faces are squares.

3. I have 2 congruent pentagons for bases. I have 5 rectangular faces.

Write the name of the solid figure.

4. 5. 6.
Draw Solid Figures from Different Views

A solid figure looks different when it is viewed from different positions.

Look at the solid figure at the right.

• There are 2 cubes in the top layer.
• There are 4 cubes in the middle layer.
• There are 6 cubes in the bottom layer.

This is a drawing of the figure viewed from the top.
This is a drawing of the figure viewed from the side.
This is a drawing of the figure viewed from the front.

For 1–6, use the figure on the right. Tell how many cubes are in each row.

1. top layer
   
2. middle layer
   
3. bottom layer
   

Draw the figure from different views.

4. top view
   
5. side view
   
6. front view
Problem Solving Skill
Make Generalizations

When you generalize, you make a statement that is true about a whole group of similar situations. Read the following problem.

Jesse is going camping. He will take with him: a box of cereal, paper towels, a flashlight, a soccer ball, and a tent shaped like a teepee. What polyhedrons will Jesse take on his camping trip?

1. Use what you know about each object to make a generalization. You can create a chart:

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>GENERALIZATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>cereal</td>
<td>Cereal usually comes in rectangular boxes.</td>
</tr>
<tr>
<td>paper towels</td>
<td>Paper towels come in a roll, which is a cylinder.</td>
</tr>
<tr>
<td>a flashlight</td>
<td></td>
</tr>
<tr>
<td>a soccer ball</td>
<td></td>
</tr>
<tr>
<td>tent</td>
<td></td>
</tr>
</tbody>
</table>

2. Sort the shapes.


4. Describe the strategy you used.

Use what you know about each object to make a generalization. Then solve.

5. Annie takes a book, 2 cans of fruit juice, and a wedge pillow to the beach. What solid figures does she have? How many of these are polyhedrons?
Customary Length

You can measure more precisely by using smaller units of measure.

Measured to the nearest inch: 2 in.

Measured to the nearest $\frac{1}{4}$ inch: $2\frac{1}{4}$ in.

Measured to the nearest $\frac{1}{16}$ inch: $2\frac{3}{16}$ in.

So, the measurement to the nearest $\frac{1}{16}$ inch is most precise.

For 1–5, use a customary ruler to measure your textbook.

1. to the nearest inch: height ______ width ______

2. to the nearest $\frac{1}{2}$ inch: height ______ width ______

3. to the nearest $\frac{1}{4}$ inch: height ______ width ______

4. to the nearest $\frac{1}{8}$ inch: height ______ width ______

5. to the nearest $\frac{1}{16}$ inch: height ______ width ______

6. Which is the most precise measure? least precise measure?

____________________________________________________________________

Measure each line segment to the nearest $\frac{1}{16}$ inch.

7. ____________________________________________________________________

8. ____________________________________________________________________

9. ____________________________________________________________________

10. ___________________________________________________________________

11. ____________________________________________________________________
Metric Length

Use your fingers to help you estimate metric length.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
</table>

Compare the width of each of your fingers to 1 centimeter. Is one of your fingers about 1 cm wide?

Use your fingers to help estimate the length of each object. Then use a ruler to measure to the nearest centimeter and millimeter.

1. 

2. 

3. 

4. 

5. 

6. 

Use a ruler to draw a line segment of the given length.

7. 6 cm 3 mm

8. 3 cm 8 mm

9. 9 cm

10. 52 mm
Change Linear Units

Use a mental image to help you decide whether to multiply or divide when changing linear units.

6 yd = □ ft  
48 in. = □ ft

Since each yard has 3 feet, multiply 6 by 3 to find the number of feet in 6 yards.

Since each foot has 12 inches, divide 48 by 12 to find the number of feet in 48 inches.

Use a mental image to help you change the units.

1. 3 ft = □ in.  
2. 12 ft = □ in.  
3. 15 km = □ m
4. 36 ft = □ yd  
5. 80 mm = □ cm  
6. 36 ft = □ in.  
7. 30 yd = □ ft  
8. 7 ft = □ in.  
9. 2 mi = □ yd

Complete.

10. 3 ft = 2 ft □ in.

3 ft = 2 ft + □ ft
= 2 ft + □ in.

11. 3 km 9 m = 2 km □ m

3 km 9 m = 2 km + □ km + 9 m
= 2 km + □ m + 9 m
= 2 km + □ m

12. 7 cm 8 mm = 6 cm □ mm

13. 8 mi 30 yd = 7 mi □ yd

Find the sum or difference.

14. 2 ft 3 in.  + 4 ft 10 in.  = □ ft □ in.

15. 2 ft 1 in.  − 9 in.  = □ ft □ in.

16. 8 m 4 cm  − 5 m 80 cm  = □ m □ cm

17. 5 m 13 cm  + 1 m 5 cm  = □ m □ cm
Customary Capacity and Weight

You can change units of weight with multiplication or division.

Change larger units to smaller units by using multiplication.

\[3 \text{ lb} = \square \text{ oz}\]

Pounds are larger than ounces, so multiply.

\[3 \times 16 = 48\]

(16 oz in 1 lb)

So, 3 lb = 48 oz.

Change smaller units to larger units by using division.

\[48 \text{ oz} = \square \text{ lb}\]

Ounces are smaller than pounds, so divide.

\[48 \div 16 = 3\]

(16 oz in 1 lb)

So, 48 oz = 3 lb.

Write multiply or divide.

1. When I change pounds to tons, I \underline{\hspace{2cm}}.

2. When I change ounces to pounds, I \underline{\hspace{2cm}}.

3. When I change tons to pounds, I \underline{\hspace{2cm}}.

4. When I change pounds to ounces, I \underline{\hspace{2cm}}.

Multiply to solve.

5. 6 lb = \underline{\hspace{2cm}} oz

6. 15 lb = \underline{\hspace{2cm}} oz

7. 4 T = \underline{\hspace{2cm}} lb

8. 1 T = \underline{\hspace{2cm}} oz

Divide to solve.

9. 20,000 lb = \underline{\hspace{2cm}} T

10. 128 oz = \underline{\hspace{2cm}} lb

11. 80 oz = \underline{\hspace{2cm}} lb

12. 14,000 lb = \underline{\hspace{2cm}} T

Multiply or divide to solve.

13. 96 oz = \underline{\hspace{2cm}} lb

14. 20 lb = \underline{\hspace{2cm}} oz

15. 5 T = \underline{\hspace{2cm}} lb

16. 12,000 lb = \underline{\hspace{2cm}} T

Customary Units for Measuring Weight

- 16 ounces (oz) = 1 pound (lb)
- 2,000 pounds = 1 ton (T)
Metric Capacity and Mass

Use the conversion table to determine whether to multiply or divide to change metric units.

Change the unit.

5 liters = □ metric cups

Think:

1 liter = 4 metric cups

Multiply by 4 to change liters to metric cups.

So, 5 liters = 20 metric cups.

Change the unit.

1. 750 mL = □ metric cups
   250 mL = _________ metric cups
   _________ by _________ to change mL to metric cups
   750 mL = _________ metric cups

2. 8.5 L = □ mL
   1 L = _________ mL
   _________ by _________ to change L to mL
   8.5 L = _________ mL

3. 5,000 g = _________ kg

4. 3 kL = _________ L

5. 7 L = _________ metric cups

6. 3,000 mg = _________ g
Time

You can calculate elapsed time by using a clock.

Think of a clock as a circular number line. Count the hours by ones and then count the minutes by fives.

Mark worked from 9 A.M. to 5:15 P.M. How many hours did Mark work?

Count the hours and then count the minutes.

So, Mark worked 8 hours 15 minutes.

Use the clocks to determine the missing information.

1. Begin A.M.  
   End P.M.  

   Elapsed time: ________________

2. Begin P.M.  
   End A.M.  

   Elapsed time: 16 hr 10 min

3. Begin A.M.  
   End P.M.  

   Elapsed time: 9 hr 45 min

4. Begin P.M.  
   End A.M.  

   Elapsed time: ________________
Problem Solving Strategy

Make a Table

You can make a table to show elapsed time.

Tara and Erin need to catch a school bus at 7:45 A.M. Before they catch their bus, they need 15 minutes to shower, 15 minutes to dress, 20 minutes to eat, and 10 minutes to walk to the bus. For what time should Tara and Erin set their alarm?

A table can help organize the information. Work backward from the final time. The starting time of one activity becomes the ending time of the previous activity.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Start Time</th>
<th>End Time</th>
<th>Elapsed Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shower</td>
<td>6:45 A.M.</td>
<td>7:00</td>
<td>15 min</td>
</tr>
<tr>
<td>Dress</td>
<td>7:00</td>
<td>7:15</td>
<td>15 min</td>
</tr>
<tr>
<td>Eat</td>
<td>7:15</td>
<td>7:35</td>
<td>20 min</td>
</tr>
<tr>
<td>Walk to bus</td>
<td>7:35</td>
<td>7:45 A.M.</td>
<td>10 min</td>
</tr>
</tbody>
</table>

So, Tara and Erin must set their alarm for 6:45 A.M.

Make a table to solve.

Chenoa is planning a hike. He will hike for 40 minutes, eat for 20 minutes, hike for 30 minutes, rest for 10 minutes, and hike for 40 minutes. He wants to end his hike at 1:30 P.M.

At what time should Chenoa start his hike? _____________
**Perimeter**

Since opposite sides of a rectangle are equal, you can use a formula to find the perimeter of a rectangle.

\[
P = (2 \times 10) + (2 \times 5)
\]

\[
P = 20 + 10
\]

\[
P = 30
\]

So, the perimeter of the rectangle is 30 yd.

Since the sides of a regular polygon are equal, you can use a formula to find the perimeter of a regular polygon.

\[
P = (\text{number of sides}) \times s
\]

\[
P = 3 \times 4
\]

\[
P = 12
\]

So, the perimeter of the triangle is 12 cm.

---

**Find the perimeter of each polygon.**

1. \(4\text{ cm}\)

   \[\text{length} = \ldots \quad \text{width} = \ldots\]

   \[
P = (2 \times \ldots) + (2 \times \ldots)
\]

   \[
P = \ldots + \ldots
\]

   \[
P = \ldots \quad \text{Perimeter is} \ldots
\]

2. \(7\text{ ft}\)

   \[\text{side} = \ldots\]

   \[
P = \ldots \times \ldots
\]

   \[
P = \ldots \quad \text{Perimeter is} \ldots
\]

3. \(3\text{ m}\)

   \[\text{side} = \ldots\]

   \[
P = \ldots \times \ldots
\]

   \[
P = \ldots \quad \text{Perimeter is} \ldots
\]

4. \(6\text{ in.}\)

   \[\text{length} = \ldots \quad \text{width} = \ldots\]

   \[
P = (2 \times \ldots) + (2 \times \ldots)
\]

   \[
P = \ldots + \ldots
\]

   \[
P = \ldots \quad \text{Perimeter is} \ldots
\]
Algebra: Circumference

The distance around a circular object is called its **circumference**.

A chord that passes through the center of a circle is a **diameter**.

If you know the diameter of a circle, you can find the circumference.

If you know the diameter of a circle, you can find the circumference.

Circumference \( \approx \) diameter \( \times \) 3.14, or \( C \approx d \times 3.14 \)

Find the circumference of this circle.

Diameter = 4

Circumference \( \approx \) diameter \( \times \) 3.14

\[ \approx 4 \times 3.14 \]

\[ \approx 12.56 \]

The circumference is approximately equal to 12.56 cm.

The diameter of each circle is given. Multiply the diameter times 3.14 to find the circumference.

1. 2. 3.
2. 3.
3. 4. 5. 6.

1. 8.2 cm
2. 7.25 cm
3. 4.3 cm

The diameter of each circle is given. Multiply the diameter times 3.14 to find the circumference.
Algebra: Area of Squares and Rectangles

You can use a formula to find the area of a rectangle.

\[ \text{Area (A)} = l \times w \]

\[ A = 6 \times 2 \]

\[ A = 12 \]

So, the area of the rectangle is 12 cm\(^2\).

You can use a formula to find the area of a square.

\[ \text{Area (A)} = s \times s \]

\[ A = 3.1 \times 3.1 \]

\[ A = 9.61 \]

So, the area of the square is 9.61 in\(^2\).

Find the area of these squares and rectangles.

1. \begin{align*}
\text{side} &= \underline{5 \text{ ft}} \\
A &= \underline{5 \times 5} \\
A &= \underline{25} \\
\text{Area is} &= \underline{25} \text{ ft}^2
\end{align*}

2. \begin{align*}
\text{length (l)} &= \underline{3 \text{ m}} \\
\text{width (w)} &= \underline{2 \text{ m}} \\
A &= \underline{3 \times 2} \\
A &= \underline{6} \\
\text{Area is} &= \underline{6} \text{ m}^2
\end{align*}

3. \begin{align*}
\text{length (l)} &= \underline{2.5 \text{ yd}} \\
\text{width (w)} &= \underline{2 \text{ yd}} \\
A &= \underline{2.5 \times 2} \\
A &= \underline{5} \\
\text{Area is} &= \underline{5} \text{ yd}^2
\end{align*}

4. \begin{align*}
\text{side} &= \underline{8.4 \text{ m}} \\
A &= \underline{8.4 \times 8.4} \\
A &= \underline{70.56} \\
\text{Area is} &= \underline{70.56} \text{ m}^2
\end{align*}
Relate Perimeter and Area

Rectangles with the same perimeter can have different areas.

Look at the rectangles below. Each rectangle has a perimeter of 24 cm, but their areas are different.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 cm</td>
<td>11 cm</td>
</tr>
<tr>
<td>B</td>
<td>2 cm</td>
<td>10 cm</td>
</tr>
<tr>
<td>C</td>
<td>3 cm</td>
<td>9 cm</td>
</tr>
<tr>
<td>D</td>
<td>4 cm</td>
<td>8 cm</td>
</tr>
<tr>
<td>E</td>
<td>6 cm</td>
<td>6 cm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle A: 11 cm²</td>
</tr>
<tr>
<td>Rectangle B: 20 cm²</td>
</tr>
<tr>
<td>Rectangle C: 27 cm²</td>
</tr>
<tr>
<td>Rectangle D: 32 cm²</td>
</tr>
<tr>
<td>Rectangle E: 36 cm²</td>
</tr>
</tbody>
</table>

Rectangle E is the rectangle with the greatest area, 36 cm².

Use the grid to draw rectangles for the given perimeter. Name the length and width of the rectangle with the greatest area.

1. Perimeter = 12 cm
   - Rectangle: 1 cm x 11 cm
   - Area: 11 cm²

2. Perimeter = 28 cm
   - Rectangle: 2 cm x 10 cm
   - Area: 20 cm²
Algebra: Area of Triangles

Use what you know about the area of a rectangle to find the area of a triangle.

- Area of a rectangle equals length × width. \( A = l \times w \)

- The area of a triangle is half the area of a rectangle with the same base and height. \( A = \frac{1}{2} \times b \times h \)

Find the area of these triangles.

1. base \( b = 3 \text{ cm} \)
   height \( h = 6 \text{ cm} \)
   \[ A = \frac{1}{2} \times 3 \times 6 = 9 \text{ cm}^2 \]
   Area is 9 cm².

2.

3.

4.

base \( b = \) ________
height \( h = \) ________
\[ A = \frac{1}{2} \times \) ________ \times \) ________
\[ A = \) ________ Area is ________.

base \( b = \) ________
height \( h = \) ________
\[ A = \frac{1}{2} \times \) ________ \times \) ________
\[ A = \) ________ Area is ________.

base \( b = \) ________
height \( h = \) ________
\[ A = \frac{1}{2} \times \) ________ \times \) ________
\[ A = \) ________ Area is ________.

base \( b = \) ________
height \( h = \) ________
\[ A = \frac{1}{2} \times \) ________ \times \) ________
\[ A = \) ________ Area is ________. 
Algebra: Area of Parallelograms

Use what you know about the area of a rectangle to find the area of a parallelogram.

- Area of a rectangle equals length \( \times \) width. \((A = l \times w)\)
- The area of a parallelogram is equal to the area of a rectangle with the same base (length) and height (width). \((A = b \times h)\)

You can use a formula to find the area of a parallelogram.

\[
\text{Area}(A) = b \times h
\]

base \((b) = 5\) cm
height \((h) = 4\) cm

\[
A = 5 \times 4
\]

\[
A = 20
\]

Area is 20 cm\(^2\).

Find the area of these parallelograms.

1. \[
\begin{align*}
\text{base} \;(b) & = \underline{8} \;\text{m} \\
\text{height} \;(h) & = \underline{2} \;\text{m}
\end{align*}
\]

\[
A = \underline{8} \times \underline{2}
\]

\[
A = 16 \quad \text{Area is} \;\underline{16}\;\text{m}^2.
\]

2. \[
\begin{align*}
\text{base} \;(b) & = \underline{5} \;\text{yd} \\
\text{height} \;(h) & = \underline{9} \;\text{yd}
\end{align*}
\]

\[
A = \underline{5} \times \underline{9}
\]

\[
A = 45 \quad \text{Area is} \;\underline{45}\;\text{yd}^2.
\]

3. \[
\begin{align*}
\text{base} \;(b) & = \underline{3} \;\text{ft} \\
\text{height} \;(h) & = \underline{1} \;\text{ft}
\end{align*}
\]

\[
A = \underline{3} \times \underline{1}
\]

\[
A = 3 \quad \text{Area is} \;\underline{3}\;\text{ft}^2.
\]

4. \[
\begin{align*}
\text{base} \;(b) & = \underline{10} \;\text{m} \\
\text{height} \;(h) & = \underline{5} \;\text{m}
\end{align*}
\]

\[
A = \underline{10} \times \underline{5}
\]

\[
A = 50 \quad \text{Area is} \;\underline{50}\;\text{m}^2.
\]
**Area of Irregular Figures**

Count whole and half square units to find the area of an irregular figure on grid paper.

Count: 8 whole squares  
12 half squares  
Divide: $12 ÷ 2 = 6$  
Add: $8 + 6 = 14$

The area of the figure is 14 square units.

When an irregular figure on grid paper has partial squares that cannot be counted exactly, use the averaging method to estimate the area.

Count: 8 whole squares  
8 partial squares  
Divide: $8 ÷ 2 = 4$  
Add: $8 + 4 = 12$

The area of the figure is about 12 square units.

**Find the area. Each square is 1 cm².**

1. 
   
   _______ whole squares  
   _______ half squares  
   _______ $÷ 2 =$ _______  
   _______ + _______ = _______  
   Area = _______ cm²

2. 
   
   _______ whole squares  
   _______ half squares  
   _______ $÷ 2 =$ _______  
   _______ + _______ = _______  
   Area = _______ cm²

**Estimate the area. Each square is 1 cm².**

3. 
   
   _______ whole squares  
   _______ partial squares  
   _______ $÷ 2 =$ _______  
   _______ + _______ = _______  
   Area is about _______ cm²

4. 
   
   _______ whole squares  
   _______ partial squares  
   _______ $÷ 2 =$ _______  
   _______ + _______ = _______  
   Area is about _______ cm²
Problem Solving Strategy

Solve a Simpler Problem

Peter wants to paint a triangle with red paint on the playground. The height of the triangle will be 20 meters and the base 5 meters. Each container of red paint covers 10 square meters. How many containers of red paint will Peter need to paint his whole triangle?

Step 1
What does the problem ask? It asks how many containers of paint Peter will need.

Step 2
Find the area of the triangle.

Area \((A) = \frac{1}{2} \times \text{base} \times \text{height}\)

\[A = \frac{1}{2} \times 5 \times 20\]

\[A = 50\] The area is 50 m\(^2\).

Step 3
Identify the number of 10 m\(^2\) containers needed to cover 50 m\(^2\).

Divide.

\[50 \div 10 = 5\]

So, Peter needs 5 containers of paint to paint the triangle on the playground.

1. Frank’s house needs new carpet. The living room is 12 feet long and 13 feet wide. The dining room is 15 feet long and 11 feet wide. How many square feet of carpet will be needed?

2. Tom is laying new sod in his yard. His front yard is 20 yd by 15 yd, and his backyard is 20 yd by 20 yd. Sod is sold by the square foot. How many square feet of sod does Tom need?
Nets for Solid Figures

A net is a two-dimensional pattern for a three-dimensional prism or pyramid.

Look at the net at the right.
- It has 1 triangular base.
- It has 3 triangular faces.

Think about how you could fold it to make a solid figure.
- It folds into a triangular pyramid.

Look at the second net at the right.
- It has 2 rectangular bases.
- It has 4 rectangular faces.

Think about how you could fold it to make a solid figure.
- It folds into a rectangular prism.

Match each solid figure with its net.

1. [Net 1] \(\rightarrow\) a.
2. [Net 2] \(\rightarrow\) b.
3. [Net 3] \(\rightarrow\) c.
Surface Area

The surface area of a solid figure is the sum of the areas of its faces.

To find the surface area of a box, add the areas of the 6 faces.

Find the area of each face. Then add the areas.

<table>
<thead>
<tr>
<th>Face</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>$5 \times 4 = 20 \text{ cm}^2$</td>
</tr>
<tr>
<td>Bottom</td>
<td>$5 \times 4 = 20 \text{ cm}^2$</td>
</tr>
<tr>
<td>Left</td>
<td>$4 \times 7 = 28 \text{ cm}^2$</td>
</tr>
<tr>
<td>Right</td>
<td>$4 \times 7 = 28 \text{ cm}^2$</td>
</tr>
<tr>
<td>Front</td>
<td>$5 \times 7 = 35 \text{ cm}^2$</td>
</tr>
<tr>
<td>Back</td>
<td>$5 \times 7 = 35 \text{ cm}^2$</td>
</tr>
<tr>
<td>Total Area</td>
<td>$166 \text{ cm}^2$</td>
</tr>
</tbody>
</table>

Use the tables to find the surface area of each box.

1. 

<table>
<thead>
<tr>
<th>Face</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>$5 \times 8 =$</td>
</tr>
<tr>
<td>Bottom</td>
<td></td>
</tr>
<tr>
<td>Left</td>
<td></td>
</tr>
<tr>
<td>Right</td>
<td></td>
</tr>
<tr>
<td>Front</td>
<td></td>
</tr>
<tr>
<td>Back</td>
<td></td>
</tr>
<tr>
<td>Total Area</td>
<td></td>
</tr>
</tbody>
</table>

2. 

<table>
<thead>
<tr>
<th>Face</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>$9 \times 2 =$</td>
</tr>
<tr>
<td>Bottom</td>
<td></td>
</tr>
<tr>
<td>Left</td>
<td></td>
</tr>
<tr>
<td>Right</td>
<td></td>
</tr>
<tr>
<td>Front</td>
<td></td>
</tr>
<tr>
<td>Back</td>
<td></td>
</tr>
<tr>
<td>Total Area</td>
<td></td>
</tr>
</tbody>
</table>
**Algebra: Volume**

**Volume** is the amount of space a solid figure occupies or can hold. The formula for volume is:

\[ \text{Volume} = \text{length} \times \text{width} \times \text{height} \]

Look at the rectangular prism at the right.

\[ V = l \times w \times h \]
\[ V = (8 \times 5) \times 6 \]
\[ V = 40 \times 6 = 240 \]

So, the volume is 240 cm\(^3\).

To find a missing dimension, use the formula for volume.

**Step 1** Substitute the known values in the formula.

\[ V = l \times w \times h \]
\[ 200 = (10 \times 4) \times h \]

**Step 2** Multiply.

\[ 200 = 40 \times h \]

**Step 3** Use Mental Math.

Think: 40 times what number equals 200?

5 = h

So, the height is 5 ft.

---

Find the volume.

1. \( l = 12 \text{ yd}, \ w = 2 \text{ yd}, \ h = 6 \text{ yd} \)
2. \( l = 7 \text{ m}, \ w = 12 \text{ m}, \ h = 2 \text{ m} \)
3. \( l = 11 \text{ cm}, \ w = 7 \text{ cm}, \ h = 3 \text{ cm} \)

Find the missing dimension.

4. length = 6 in.
   width = 8 in.
   height = ______
   Volume = 240 in.\(^3\)
5. length = ______
   width = 5 m
   height = 2 m
   Volume = 150 m\(^3\)
6. length = 10 ft
   width = ______
   height = 9 ft
   Volume = 270 ft\(^3\)
7. length = 4 ft
   width = 5 ft
   height = ______
   Volume = 120 ft\(^3\)
8. length = 6 cm
   width = ______
   height = 12 cm
   Volume = 216 cm\(^3\)
9. length = ______
   width = 14 in.
   height = 7 in.
   Volume = 1,960 in.\(^3\)
Measure Perimeter, Area, and Volume

Keywords can help you decide upon the appropriate unit of measure.

• Use *units* to measure the length of or distance around an object.
  
  Keywords: around, length, height, distance, perimeter

• Use *square units* to measure the area of an object.
  
  Keywords: cover, area, surface area

• Use *cubic units* to measure the volume of an object.
  
  Keywords: volume, capacity, fill, space

Substitute *inches, meters,* and so on for *units* when you are given specific measurements.

---

Underline the keywords and tell the appropriate units to measure each. Write *units, square units,* or *cubic units.*

1. capacity of a mug
2. paper to cover a box
3. length of a room

---

Underline the keyword and write the units you would use to measure each.

4. surface area of this cube
   
   ![Diagram of a cube](4 ft)

5. perimeter of this square
   
   ![Diagram of a square](8 in.)

6. volume of this prism
   
   ![Diagram of a prism](8 cm, 7 cm, 3 cm)

7. area of this parallelogram
   
   ![Diagram of a parallelogram](2 m, 4 m)
Problem Solving Skill

Use A Formula

Paul wants to send some things to his brother at camp. He finds a box in his garage that is 12 inches long and 10 inches wide. It has a volume of 1,200 cubic inches. Paul wants to pack a container that is 14 inches high. Will the container fit in the box?

To answer the question, you need to find the height of the box.

Use the formula for volume. \( V = l \times w \times h \)

- Substitute. \( 1,200 = 12 \times 10 \times h \)
- Multiply. \( 1,200 = 120 \times h \)
- Use Mental Math. \( h = 10 \)

The height is 10 inches. The answer is no, the container will not fit in the box.

Use a formula and solve.

1. Rita wants to put a wallpaper border around her room. Her room is 11 ft by 13 ft. How many ft of border does she need to do the job? How many sq ft of carpet are required to carpet the room?

2. Matthew needs to return a lamp that measures 15 inches by 12 inches by 8 inches. He has a box that is 16 inches long and 13 inches wide. It has a volume of 2,080 cubic inches. Is the box big enough for the lamp?

3. Mark needs to ship 500,000 cm\(^3\) cubic centimeters of peanuts. The shipping crate is 90 centimeters by 80 centimeters by 70 centimeters. Is it large enough to ship the peanuts? Explain.

4. How many square feet of carpet do you need to cover a 12-foot by 15-foot room? how many square yards?
**Understand Ratios**

You can use decimal models to help find ratios. Ratios compare two quantities. There are three types of ratios.

### Part to Whole

**Shaded parts:** 4  
**Total parts:** 10  
So, the ratio of part to whole is 4 to 10.

### Whole to Part

**Total parts:** 10  
**Shaded parts:** 6  
So, the ratio of whole to part is 10 to 6.

### Part to Part

**Shaded parts:** 3  
**Unshaded parts:** 7  
So, the ratio of part to part is 3 to 7 or 7 to 3.

---

**Complete the ratios.**

1. **Shaded parts:**  
   **Total parts:**  
   **Part to whole ratio:**

2. **Shaded parts:**  
   **Unshaded parts:**  
   **Part to part ratio:**

3.  
   **Part to whole ratio:**  
   **Whole to part ratio:**  
   **Part to part ratio:**

4.  
   **Part to whole ratio:**  
   **Whole to part ratio:**  
   **Part to part ratio:**

**Tell which type of ratio is expressed.**

5.  
   5 to 5  
   10 to 5
Express Ratios

You can write ratios in three ways.

A **part to whole** ratio can be written:

- 6 to 10, 6:10, \( \frac{6}{10} \)

A **whole to part** ratio can be written:

- 10 to 6, 10:6, \( \frac{10}{6} \)

A **part to part** ratio can be written:

- 6 to 4, 6:4, \( \frac{6}{4} \)

Write each ratio in three ways. Then name the type of ratio.

1. 4 red counters to 3 green counters
   - 4:3, \( \frac{4}{3} \)
   - 3:4, \( \frac{3}{4} \)

2. 12 pencils to 6 pens
   - 12:6, \( \frac{12}{6} \)
   - 6:12, \( \frac{6}{12} \)

3. 2 soccer balls out of 11 balls
   - 2:11, \( \frac{2}{11} \)
   - 11:2, 11:2

4. 16 of 24 students are boys
   - 16:24, \( \frac{16}{24} \)
   - 24:16, \( \frac{24}{16} \)

Write a, b, or c to show which ratio represents each comparison.

5. 3 red apples out of 8 apples
   - a \( \frac{3}{8} \), b 3:8, c 3 to 8

6. 7 boys to 8 girls
   - a \( \frac{7}{8} \), b 7:8, c 7 to 8

7. 8 baseballs to 13 basketballs
   - a 8:13, b 13:8, c \( \frac{8}{13} \)

8. 1 month out of 12 months
   - a 1:12, b 12 to 1, c 1:12

Write each ratio in two other ways.

9. 3:5
   - \( \frac{3}{5} \), 15:25

10. 11 to 13
    - \( \frac{11}{13} \), 22:26

11. 28 to 47
    - \( \frac{28}{47} \), 47:28

12. 14:6
    - \( \frac{14}{6} \), 7:3

13. \( \frac{21}{4} \)
    - 21:4, 10.5 to 2

14. \( \frac{7}{19} \)
    - 7:19, 14:38
Ratios and Proportions

You can use pictures to show equivalent ratios.

This shows the ratio 3:4. This shows an equivalent ratio, 6:8.

\[
\begin{align*}
\text{□□□□} & \quad \text{△△△△} \\
\text{△△△△} & \quad \text{□□□□ □□□□}
\end{align*}
\]

You can also use fractions to show equivalent ratios.

\[
\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24}
\]

Draw pictures to determine if the ratios are equivalent. Then write yes or no.

1. 1:2 and 3:6  
   2. 3:4 and 4:6  
   3. 2:3 and 3:5  
   4. 2:5 and 4:10

Write two fractions that are equivalent to each ratio.

5. \( \frac{4}{5} = \quad = \quad \)
6. \( \frac{9}{2} = \quad = \quad \)
7. \( \frac{11}{12} = \quad = \quad \)
8. \( \frac{6}{10} = \quad = \quad \)
9. \( \frac{8}{6} = \quad = \quad \)
10. \( \frac{3}{7} = \quad = \quad \)

Write two ratios that are equivalent to each ratio.

11. 2:3  
    12. 3 to 4  
13. 8 to 12  
    14. 5:7  
15. 11:9  
    16. 16 to 4  
17. 1:6  
    18. 2 to 10  
19. 7:11  
    20. 2:6
Scale Drawings

You can use map scales and equivalent ratios to determine actual distances.

In this map of New Jersey, the scale is 4 mm = 1 mi. So, the ratio of millimeters to miles is 4:1.

The map distance from Paterson to Leonia is about 36 mm. What is the actual distance in miles?

Use equivalent ratios.

\[
\frac{4}{1} = \frac{36}{n} \quad \text{left} \quad 4 \times 9 = 36 \\
\frac{1}{n} = \frac{1 \times 9}{9}
\]

Since \(4 \times 9 = 36\), you would multiply \(1 \times 9\).

So, the distance from Paterson to Leonia is 9 miles.

Use the map scale above and equivalent ratios to find the actual distance.

1. The map distance from Paterson to Newark is 48 mm.

What is the distance in miles? _________

2. The map distance from Montclair to Orange is 12 mm.

What is the distance in miles? _________

3. The map distance from Paterson to Orange is 42 mm.

What is the distance in miles? _________

Use a map scale of 1 cm = 15 mi and equivalent ratios to complete the table.

<table>
<thead>
<tr>
<th>4. Watertown to Belmont</th>
<th>6 cm</th>
<th>_________</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Arlington to Bedford</td>
<td>4 cm</td>
<td>_________</td>
</tr>
<tr>
<td>6. Belmont to Avon</td>
<td>11 cm</td>
<td>_________</td>
</tr>
<tr>
<td>7. Franklin to Millis</td>
<td>3.5 cm</td>
<td>_________</td>
</tr>
</tbody>
</table>
Problem Solving Skill

Too Much/Too Little Information

Sometimes you have *too much* or *too little* information to solve a problem. When you are given *too much* information, you must decide what information to use to solve the problem. When you are given *too little* information, you can’t solve the problem.

Read the table carefully. Look at the question and decide if you have *too much* or *too little* information.

What is Jared’s ratio of rainbowfish to tetras?

What information you **Know:**
- You know Jared’s ratio of fish to rainbowfish is 25:3.
- You know Jared’s ratio of fish to tetras is 25:5.

What information you **Don’t Need:**
- You don’t need the information on catfish.

You have too much information, so you can solve the problem. Jared’s ratio of rainbowfish to tetras is 3:5.

---

Jared’s Fish Populations

<table>
<thead>
<tr>
<th>Fish to Types of Fish</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>fish : catfish</td>
<td>5:1</td>
</tr>
<tr>
<td>fish : rainbowfish</td>
<td>25:3</td>
</tr>
<tr>
<td>fish : tetras</td>
<td>25:5</td>
</tr>
</tbody>
</table>

Use the table to complete each problem.

1. How many red rainbowfish does Jared have for every one rainbowfish?
   - **What you Know:**
   - **What you Don’t Need:**
   - **What you Need to Know:**

2. How many fish are there for every one tetra?
   - **What you Know:**
   - **What you Don’t Need:**
   - **What you Need to Know:**
Understand Percent

You can represent part of the whole by using percents. **Percent** means “per hundred.” 100 percent is the whole.

The $10 \times 10$ grid has 100 squares. Each square represents 1 percent.

33 squares are shaded. So, 33% of the squares are shaded. 67% of the squares are unshaded.

56 squares are shaded. So, 56% of the squares are shaded. 44% of the squares are unshaded.

Write the percents for the shaded and unshaded squares.

1. 2. 3.

Percent shaded ___  Percent shaded ___  Percent shaded ___

Percent unshaded ___  Percent unshaded ___  Percent unshaded ___

Shade the $10 \times 10$ grid to show the percent.

4. 34%  5. 69%  6. 82%
Relate Decimals and Percents

Percents and decimals both represent a part of a whole, or of 100. You can use money to compare percents and decimals.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Dime</th>
<th>Nickel</th>
<th>Penny</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimal:</td>
<td>$0.25</td>
<td>$0.10</td>
<td>$0.05</td>
</tr>
<tr>
<td>Read:</td>
<td>twenty-five hundredths</td>
<td>ten hundredths</td>
<td>five hundredths</td>
</tr>
<tr>
<td>Ratio:</td>
<td>25 out of 100</td>
<td>10 out of 100</td>
<td>5 out of 100</td>
</tr>
<tr>
<td>Percent:</td>
<td>25% of a dollar</td>
<td>10% of a dollar</td>
<td>5% of a dollar</td>
</tr>
</tbody>
</table>

Write a decimal and a percent to describe each total amount.

1. 1 quarter, 2 dimes
   - decimal _______
   - percent _______

2. 1 quarter, 1 dime, 1 penny
   - decimal _______
   - percent _______

3. 3 quarters, 3 pennies
   - decimal _______
   - percent _______

4. 8 dimes, 3 nickels, 2 pennies
   - decimal _______
   - percent _______

5. 12 nickels, 4 pennies
   - decimal _______
   - percent _______

6. 3 pennies
   - decimal _______
   - percent _______

Write the number as a decimal and as a percent.

7. forty-five hundredths
   - __________________________
   - __________________________

8. twenty-one hundredths
   - __________________________
   - __________________________

9. eighty-four hundredths
   - __________________________
   - __________________________

10. seventy-two hundredths
   - __________________________
   - __________________________
Relate Fractions, Decimals, and Percents

Percents can be written as decimals, or as fractions with 100 as the denominator.

• 45% means forty-five hundredths, or 0.45.

• 45% also means \( \frac{45}{100} \).

To write a fraction in simplest form, divide the numerator and the denominator by the same number. Keep doing this until 1 is the only common factor.

\( 45\% = \frac{45}{100} = \frac{45 \div 5}{100 \div 5} = \frac{9}{20} \)

So, 45% = 0.45 = \( \frac{9}{20} \).

To write a fraction as a percent, write a fraction with the percent as the numerator and 100 as the denominator.

\( \frac{37}{100} = 37\% \), or 0.37

\( \frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 75\% \), or 0.75

Complete. Write each as a decimal, a percent, and a fraction in simplest form.

1. 0.35 = 35% = \( \frac{35}{100} \) = \( \frac{35 \div 5}{100 \div 5} \) = _______

2. _______ = 25% = \( \frac{25}{100} \) = \( \frac{25 \div 25}{100 \div 25} \) = _______

3. _______ = 20% = \( \frac{20}{100} \) = \( \frac{20 \div 20}{100 \div 20} \) = _______

4. 0.90 = _______ = \( \frac{90}{100} \) = \( \frac{90 \div 10}{100 \div 10} \) = _______

5. 0.16 = _______ = \( \frac{16}{100} \) = \( \frac{16 \div 16}{100 \div 16} \) = _______

6. 0.49 = 49% = \( \frac{49}{100} \) = \( \frac{49 \div 10}{100 \div 10} \) = _______

Complete. Write as a decimal and as a percent.

7. \( \frac{1}{20} \) = \( \frac{1}{100} \) = _______, or _______ 8. \( \frac{3}{10} \) = \( \frac{3}{100} \) = _______, or _______

9. \( \frac{11}{100} \) = \( \frac{11}{100} \) = _______, or _______ 10. \( \frac{6}{25} \) = \( \frac{6}{100} \) = _______, or _______

11. \( \frac{2}{5} \) = \( \frac{2}{100} \) = _______, or _______ 12. \( \frac{3}{4} \) = \( \frac{3}{100} \) = _______, or _______
Find a Percent of a Number

You can make a model to find a percent of a number.

Find 40% of 30.

**Step 1**
Use pieces of paper to represent 30.

**Step 2**
Separate the pieces of paper into 10 equal groups. Each group represents 10%.

**Step 3**
Separate 4 groups from the rest. These 4 groups represent 40%.

Since each group has 3 pieces of paper, 4 groups have 12 pieces of paper. So, 40% of 30 equals 12.

You can find a percent of a number by changing the percent to a decimal and multiplying.

**Step 1**
Change the percent to a decimal.

40% = 0.40

**Step 2**
Multiply the number by the decimal.

0.40 × 30 = 12

So, 40% of 30 equals 12.

Use a decimal to find the percent of the number.

1. 10% of 30
2. 20% of 50
3. 15% of 40
4. 20% of 60
5. 25% of 40
6. 3% of 18
Mental Math: Percent of a Number

You can use mental math to find a percent of a number.

Find 30% of 50 chips

Think: 30% = 10% + 10% + 10%

\[30\% \text{ of } 50 = (10\% \times 50) + (10\% \times 50) + (10\% \times 50)\]

\[= (0.1 \times 50) + (0.1 \times 50) + (0.1 \times 50)\]

\[= 5 + 5 + 5 = 15\]

So, 30% of 50 = 15.

Use mental math to find the percent of each number.

1. 60% of 20
2. 80% of 30
3. 25% of 60

4. 15% of 40
5. 70% of 50
6. 45% of 20

7. 130% of 4,000
8. 20% of 150
9. 10% of 2,000

10. 70% of 80
11. 15% of 20
12. 200% of 10,000

13. 85% of 100
14. 60% of 500
15. 70% of 800
**Problem Solving Strategy**

**Make a Graph**

You can make a graph to display percent data.

**Step 1** Review your data. If your data show the relationship of parts to a whole, you can use a circle graph.

**Step 2** Divide a circle into 10 equal sections.

**Step 3** Label the number of sections that show each percent.

1 section represents 10%.
2 sections represent 20%.
There are three 20% sections.
3 sections represent 30%.

**Step 4** Label the percents and title the circle graph.

Use a 10-section circle and the data in the table to make a circle graph.

<table>
<thead>
<tr>
<th>FAVORITE MAGAZINES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magazine</td>
</tr>
<tr>
<td>Sports Illustrated for Kids</td>
</tr>
<tr>
<td>National Geographic</td>
</tr>
<tr>
<td>Nickelodeon</td>
</tr>
<tr>
<td>Zoo Zillions</td>
</tr>
<tr>
<td>People</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FAVORITE VACATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacation Place</td>
</tr>
<tr>
<td>National Park</td>
</tr>
<tr>
<td>Beach</td>
</tr>
<tr>
<td>Amusement Park</td>
</tr>
<tr>
<td>Foreign Country</td>
</tr>
<tr>
<td>Famous City</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FAVORITE HOBBIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hobby</td>
</tr>
<tr>
<td>Painting</td>
</tr>
<tr>
<td>Collecting Stamps</td>
</tr>
<tr>
<td>Making Models</td>
</tr>
<tr>
<td>Collecting Stuffed Animals</td>
</tr>
<tr>
<td>Other</td>
</tr>
</tbody>
</table>
**Compare Data Sets**

You can find a percent of a number to compare the results of two or more sets of data.

The circle graphs below show the results of surveys Joshua conducted.

**Joshua's family**
There were 20 people surveyed.

**Joshua's neighborhood**
There were 60 people surveyed.

Joshua wants to know in which survey vanilla got more votes.

<table>
<thead>
<tr>
<th>Joshua's family</th>
<th>Joshua's neighborhood</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FAVORITE ICE-CREAM FLAVORS</strong></td>
<td><strong>FAVORITE ICE-CREAM FLAVORS</strong></td>
</tr>
<tr>
<td>in Joshua's family</td>
<td>in Joshua's neighborhood</td>
</tr>
<tr>
<td>Chocolate: 45%</td>
<td>Chocolate: 40%</td>
</tr>
<tr>
<td>Vanilla: 30%</td>
<td>Vanilla: 20%</td>
</tr>
<tr>
<td>Mint Chip: 10%</td>
<td>Mint Chip: 15%</td>
</tr>
<tr>
<td>Other: 10%</td>
<td>Other: 10%</td>
</tr>
<tr>
<td><strong>Step 1</strong></td>
<td><strong>Step 1</strong></td>
</tr>
<tr>
<td>Change the percent to a decimal.</td>
<td>Change the percent to a decimal.</td>
</tr>
<tr>
<td>30% = 0.30</td>
<td>20% = 0.20</td>
</tr>
<tr>
<td><strong>Step 2</strong></td>
<td><strong>Step 2</strong></td>
</tr>
<tr>
<td>Multiply the total number of people by the decimal.</td>
<td>Multiply the total number of people by the decimal.</td>
</tr>
<tr>
<td>0.30 × 20 = 6</td>
<td>0.20 × 60 = 12</td>
</tr>
</tbody>
</table>

So, 6 people voted for vanilla.

So, vanilla received more votes in Joshua's neighborhood survey.

For 1–2, use the circle graphs above.

1. In which survey did chocolate receive more votes? How many more votes?

2. If vanilla had received only 10% of the votes in Joshua's neighborhood, in which survey would vanilla have received more votes? Explain.
Probability Experiments

A box contains 6 black marbles and 2 white marbles.

Experiment: Shake the box, and pull a marble. Record the color; then replace the marble.

There are two possible events for this experiment.

- The marble is black.
- The marble is white.

Tom conducts the experiment 16 times. He predicts that the events will be 12 black and 4 white marbles. He records the actual results in a table.

<table>
<thead>
<tr>
<th>MARBLE EXPERIMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Events</td>
</tr>
<tr>
<td>Predicted frequency</td>
</tr>
<tr>
<td>Actual frequency</td>
</tr>
</tbody>
</table>

1. Is white or black a more likely event? Why?

2. What were the actual frequencies after 16 trials?

3. Explain why Tom predicted 12 black and 4 white marbles.
Outcomes

Your cafeteria offers a choice of tuna, turkey, or veggie sandwiches. You can also choose between white and wheat bread. What are your possible choices?

A tree diagram shows you all the possible choices.

<table>
<thead>
<tr>
<th>Breads</th>
<th>Sandwiches</th>
<th>Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>white</td>
<td>tuna</td>
<td>tuna sandwich on white bread</td>
</tr>
<tr>
<td></td>
<td>turkey</td>
<td>turkey sandwich on white bread</td>
</tr>
<tr>
<td></td>
<td>veggie</td>
<td>veggie sandwich on white bread</td>
</tr>
<tr>
<td>wheat</td>
<td>tuna</td>
<td>tuna sandwich on wheat bread</td>
</tr>
<tr>
<td></td>
<td>turkey</td>
<td>turkey sandwich on wheat bread</td>
</tr>
<tr>
<td></td>
<td>veggie</td>
<td>veggie sandwich on wheat bread</td>
</tr>
</tbody>
</table>

So, you have 6 possible choices.

1. Amanda must choose swimming a 25-meter, 50-meter, or 100-meter race. She can swim in either the freestyle or backstroke division. How many choices does Amanda have? Complete the tree diagram.

<table>
<thead>
<tr>
<th>Divisions</th>
<th>Races</th>
<th>Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>freestyle</td>
<td>25-meter</td>
<td>25-meter freestyle</td>
</tr>
<tr>
<td></td>
<td>50-meter</td>
<td>____________________</td>
</tr>
<tr>
<td></td>
<td>___________</td>
<td>____________________</td>
</tr>
<tr>
<td>backstroke</td>
<td>___________</td>
<td>____________________</td>
</tr>
<tr>
<td></td>
<td>100-meter</td>
<td>_______________</td>
</tr>
</tbody>
</table>

Amanda has _____ choices.

2. Brian’s parents are buying a new car. They can choose a sedan or a minivan. Both cars are available in red or white. How many choices do they have? ________
Probability Expressed as a Fraction

You can predict the probability, or chance, that an event will happen.

Ben has a spinner with six sections. The possible outcomes are spinning blue, spinning red, spinning yellow, or spinning green. What is the probability of spinning blue?

Probability of spinning blue = \( \frac{\text{number of blue sections}}{\text{total number of sections}} = \frac{2}{6} \)

So, the probability of spinning blue is \( \frac{2}{6} \), or \( \frac{1}{3} \).

For Problems 1–4, use spinner A. Give the probability of spinning each color.

1. blue \( \text{___________} \)
2. red \( \text{___________} \)
3. green \( \text{___________} \)
4. yellow \( \text{___________} \)

For Problems 5–8, use spinner B. Give the probability of spinning each number.

5. 1 \( \text{___________} \)
6. 2 \( \text{___________} \)
7. 3 \( \text{___________} \)
8. 4 \( \text{___________} \)
9. 5 \( \text{___________} \)
Compare Probabilities

You can compare the probabilities of events to determine whether one event is more likely than another.

Kim has a bag of marbles with 6 blue and 3 green marbles. Which color marble is she more likely to pull?

First, find the probability of each event.

Probability of blue = \( \frac{\text{number of blue marbles}}{\text{total number of marbles}} = \frac{6}{9} \)

Probability of green = \( \frac{\text{number of green marbles}}{\text{total number of marbles}} = \frac{3}{9} \)

Then, compare the probabilities.

\( \frac{6}{9} > \frac{3}{9} \)

So, Kim is more likely to pull a blue marble.

For Problems 1–2, use the spinner. Find the probability of each event. Decide which event is more likely.

1. The pointer will land on blue; the pointer will land on red.

Probability of blue = \( \frac{\text{number of blue sections}}{\text{total number of sections}} = \frac{2}{8} \)

Probability of red = \( = \)

More likely event: ________

2. The pointer will land on yellow; the pointer will land on green.

Probability of ________ = \( = \)

Probability of ________ = \( = \)

More likely event: ________
Problem Solving Strategy

Make an Organized List

Making an organized list can help you determine the possible outcomes of a probability experiment.

Sharon has a coin and a spinner divided into two sections: red and yellow. She will toss the coin and spin the spinner. What are the possible outcomes? How many are there?

<table>
<thead>
<tr>
<th>Spinner</th>
<th>Coin</th>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>heads</td>
<td>red and heads</td>
</tr>
<tr>
<td></td>
<td>tails</td>
<td>red and tails</td>
</tr>
<tr>
<td>yellow</td>
<td>heads</td>
<td>yellow and heads</td>
</tr>
<tr>
<td></td>
<td>tails</td>
<td>yellow and tails</td>
</tr>
</tbody>
</table>

So, there are 4 possible outcomes.

Make an organized list to solve.

1. Jereme is conducting a probability experiment with a coin and a bag of marbles. He has 3 marbles in the bag: 1 red, 1 purple, and 1 brown. He will replace the marble after each turn. How many possible outcomes are there for this experiment? What are they?

<table>
<thead>
<tr>
<th>Marbles</th>
<th>Coin</th>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>heads</td>
<td>red and heads</td>
</tr>
<tr>
<td></td>
<td>tails</td>
<td></td>
</tr>
<tr>
<td>purple</td>
<td></td>
<td></td>
</tr>
<tr>
<td>brown</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There are ______ possible outcomes.

2. Sarah has 10¢. How many different combinations of coins could she have? ____________________________